Hierarchical Structures and Dynamic Incentives*

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ABSTRACT

We study the optimal hierarchical structure of organizations under limited commitment. In our model, the organization can commit to its hierarchical structure, but not to a long–term wage. We show that the optimal hierarchy is *horizontal* only for fast growing organizations with small uncertainty about the employees' efficiency. When uncertainty is large or the organization's growth is slow, the optimal hierarchy is *vertical*. In particular, vertical hierarchies ease tension between efficient and inefficient agents' dynamic incentives, which facilitates information revelation. Our study also indicates that seniority rather than merit is decisive for promotion, and production complementarity favors vertical hierarchies.

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1 Introduction

Since the seminal work of Spence (1973) on job market signaling, economists have recognized the importance of private information in organizations. Extending the insight to dynamic settings, subsequent literature shows that dynamic incentives for information revelation play a crucial role when the organization's commitment is limited. The complicating matter being that, unlike in the case of full commitment, the organization under limited commitment must motivate its employees in a time consistent manner.

Focusing on the need for time consistency and dynamic incentives, we develop a theory of optimal hierarchical design, following the view of organization theory that a hierarchical structure defines authorities and information flows within the organization.[1] Using a dynamic agency model, we analyze why and under what circumstances one hierarchical structure prevails over the other — the horizontal hierarchy with flat and centralized structure, or the vertical hierarchy with longer and more restricted communication lines.

Our analysis reveals that the horizontal hierarchy is optimal when the organization's output expands quickly over time and the uncertainty about the efficiency of its potential workers is small. By contrast, when such uncertainty is large or output growth is relatively slow, the vertical hierarchy is optimal. This result is consistent with stylized observations that fast growing organizations with a highly efficient workforce typically exhibit flatter, more horizontal hierarchies, whereas organizations in more mature industries that grow less quickly and pick their workers from an average pool tend to have vertical hierarchies.

Our framework considers a growing organization with limited commitment — the organization can commit to its hierarchical structure, but not to a long–term labor contract.[2] In line with Laffont and Tirole (1987, 1988), such a limited commitment leads to a tension between the dynamic incentives of an efficient and an inefficient agent. With regard to an *efficient* agent, the organization has to take care of a *hold–up* problem. Due to the organization's lack of commitment, an efficient agent who reveals his high efficiency in the earlier period fears exploitation and forfeited rents in the future. In order to circumvent this

[1] E.g., Laffont and Martimort (1998), Rajan and Zingales (2001), and Friebel Raith (2004).

[2] Empirical evidence (e.g., Markoczy 2001) suggest that, once in place, the organizational structure is indeed difficult to change, even if this is in the ex post interest of the organization. On the other hand, the commitment power of wage contracts is, for instance, typically limited by law due to non–slavery clauses or other unalienable worker's rights. In addition, fully fledged wage schemes are complex, and thus often subject to changes in the future, while an organization's hierarchical structure is hard to change once set up.

hold–up problem, the organization needs to pay these future rents to the efficient agent as a high wage up–front. But such a high up–front wage then creates another incentive problem with regard to the *inefficient* agent — the *take–the–money–and–run* problem.[3] That is, an efficient agent will misrepresent himself in order to "take" the high wage in the earlier period, and subsequently "run" by rejecting the organization's offer in the later period.

We study how the organization's hierarchical structure affects the tension between these two dynamic incentive problems and show that, in comparison to the horizontal hierarchy, the vertical hierarchy relaxes it, although at a cost. In particular, the vertical hierarchy has two indirect strategic effects: a *carrot effect* which relaxes the take–the–money–and–run problem associated with an inefficient agent, and a *play–hard–ball effect* which relaxes the hold–up problem associated with an efficient agent.

First, the "carrot" effect allows the organization to pledge rents to an agent in the future. In the vertical hierarchy, the agent will have exclusive communication channels in the later period, which is the source of his future rents. Thus, if an *inefficient* agent decides to mimic an efficient agent in the earlier period and "run," then he will lose his rents in the later period. Consequently, the vertical hierarchy discourages the inefficient agent from misrepresenting his type in the earlier period to play the take–the–money–and–run strategy.

Second, the "play–hard–ball" effect allows the organization to lower output credibly in later periods. Due to the loss of control in the vertical hierarchy, the cost of operation is higher, and this makes the organization's future production cuts more credible. This implies that an *efficient* agent's future rents from his private information becomes smaller. Consequently, such smaller rents in the later period reduces the agent's reluctancy to reveal his information in the earlier period. In other words, the efficient agent's misrepresenting incentive associated with the hold–up problem becomes less severe.

Although the vertical hierarchy has the aforementioned two strategic effects, its optimality depend on the underlying parameters of the model, in particular, the uncertainty about the efficiency of the agents. We show that the vertical hierarchy mitigates the take–the–money–and–run problem when inefficient agents are relatively likely, while it mitigates the hold–up problem, when efficient agents are relatively likely. When the efficiency uncertainty about agents is large, the vertical hierarchy exhibits both effects and is especially effective. Hence, the vertical hierarchy is beneficial when information revelation is important.

Interpreting the vertical hierarchy as an organizational structure that promotes a senior

[3] See Laffont and Tirole (1993, p.377) and Bolton and Dewatripont (2005, p.395) for this terminology.

member to middle management yields additional insights. Under this interpretation, the carrot effect relates the pecuniary benefits of middle management to their privileged position to control communication streams and thereby reap rents from their acquired information about their subordinates.[4] In addition, we show in an extension that the unconditional vertical hierarchy, which promotes the senior agent regardless of his efficiency, dominates a conditional hierarchy, which promotes only the efficient agents. This result provides a rationale for the role of seniority with regard to promotions in an organization. The explanation is a direct consequence of the strategic carrot effect in the vertical hierarchy — promotions based on seniority reduces the take–the–money–and–run by the *less efficient* employees, and tie them to the organization, which in turn, facilitates revelation of useful information.

In the final extension, we consider an alternative production function, with which one agent's production does not affect the other agent's production. Comparing optimal hierarchal structures under different production functions reveals that, while complementarity in production is not crucial for our main result, it favors vertical hierarchies. It leads us to the empirical prediction that vertical hierarchies are more common in organizations where the production process is more complementary.

The rest of the paper is organized as follows. The next section discuss the related literature. Section 3 present the formal framework. In Section 4, we analyze the horizontal hierarchy in which each agent has direct access to the principal. The vertical hierarchy in which the principal only deals with the senior agent is discussed in Section 5. The extensions of our analysis are discussed Section 6. We conclude the paper with remarks in Section 7. All proofs are relegated to Appendix 1. In Appendix 2, we demonstrate that our results are robust when considering semi–separating strategies.

2 Related Literature

Among the many contributions about hierarchical structures,[5] the following studies are more related to ours. Laffont and Martimort (1998) show that the vertical hierarchy enables the

[4] This is in line with frequent complaints among junior workers of an organization that their superiors often take credits for their ideas and achievements. See, for instance, Bartolome (1989) for this issue.

[5] See, for example, Williamson (1967) and McAfee and McMillan (1995) that advocate the horizontal hierarchy by identifying loss of control in the vertical hierarchy. Calvo and Wellisz (1979) and Qian (1994) analyze optimal wage transfers with respect to hierarchical ranks. Rosen (1982), Harris and Raviv (2002), and Hart and Moore (2005) study coordination issues in different hierarchical structures.

organization to effectively discriminate wage transfers among the agents of different efficiencies, thus mitigating the agents' incentives for collusion. Rajan and Zingales (2001) demonstrate that the horizontal hierarchy prevents the employees from taking the organization's production technology and becoming a competitor, while the vertical hierarchy provides employees with strong incentives to learn the tasks their immediate superior assigns them. They show that the prevailing hierarchy is horizontal when the organization's production is labor–intensive, and vertical when production is capital–intensive. Using a moral hazard framework, Friebel and Raith (2004) show that exclusive communication lines to top management provide middle managers with appropriate incentives to make a sincere effort for recruiting and training subordinates. Unlike these papers, our study shows how an organization uses its hierarchical structure to cope with dynamic incentive problems associated with hidden information and limited commitment.

In a static model, Baron and Besanko (1992) show that if top management can monitor the transaction between the subunits, then the vertical hierarchy is equivalent to the horizontal hierarchy. Melumad et al. (1992) breaks this tie between the horizontal and vertical hierarchy and show that the vertical hierarchy dominates the horizontal hierarchy in the presence of costly communication. Stressing commitment problems in dynamic setups, Crémer (1995) demonstrate that an organization may benefit from more indirect communication lines that limit information flows and keep the employee at a distance, because it makes the organization's punishment more credible. Aghion and Tirole (1997) show that providing a subordinate with authority induces the subordinate to acquire and provide useful information for the organization. Dessein (2002) demonstrates that delegation of control to a subordinate improves his incentives to provide information, when the organization's commitment is limited. The last three papers, while dealing with commitment problems, they do not focus on the optimal hierarchical structure of multi–layered organizations.

The organization literature recognizes the incentive effect of promotions mostly from a hidden action perspective. Starting with Lazaer and Rosen (1981), an extensive literature points out that promotions can serve as tournaments among agents to improve incentives. In contrast, our incentive problem originates from hidden information rather than hidden action. As a consequence, our framework allows us to incorporate the source of rent after promotion into our analysis. Moreover, practice of seniority presents a puzzle to the tournament literature, in which promotions are handed out primarily on the basis of merit. In our model, seniority arises as an optimal structure of an organization.

3 The Framework

We consider a two period ($t = 1, 2$) model in which the top management of the organization (the principal) potentially hires two experts (the agents). We assume that the organization's revenue opportunities and available technologies grow over time. For this reason, the principal hires one agent (agent A) in $t = 1$, and hires an additional agent (agent B) in $t = 2$. The output level of agent A in $t = 1$ is denoted by $q_{A1} \in \{0, 1\}$.[6] Because output level is verifiable, the first period contract is a wage ω_1 from the principal to agent A contingent on his output level. In $t = 2$, agent A and B produce $q_{A2} \in \{0, \gamma\}$ and $q_{B2} \in \{0, \gamma\}$ respectively, where $\gamma > 1$ represents the organization's output expansion parameter.

The agent's individual output levels map into the organization's revenue as follows. In $t = 1$, the organization's revenue, q_1, is simply agent A's output level q_{A1}, and thus:

$$q_1 = q_{A1} \in \{0, 1\}.$$

In $t = 2$, the agents' productions, $q_{A2} \in \{0, \gamma\}$ and $q_{B2} \in \{0, \gamma\}$, are complementary, and the organization's revenue is:

$$q_2 = \begin{cases} 2\gamma & \text{if } q_{A2} = q_{B2} = \gamma, \\ 0 & \text{otherwise.} \end{cases}$$

The complementarity assumption represents the gain of specialization in an expanding organization that conducts overall production in specific tasks in which different agents specialize.[7] In $t = 2$, each agent can produce the output $\gamma > 1$ but final output is realized only if both tasks are performed. In contrast, in $t = 1$ agent A performs both tasks so that we can express output levels by a one–dimensional $q_1 \in \{0, 1\}$. For simplicity, we normalize the common discount factor to 1, and therefore the organization's total revenue over the two periods is given by:[8]

$$Q = q_1 + q_2.$$

An agent $i \in \{A, B\}$ incurs the cost of production:

$$C(q_{it}, \theta) = (1 - \theta)q_{it},$$

[6] We employ a binary setup primarily to avoid issues of non–tractability for which dynamic adverse selection models with limited commitment are well known (e.g., Laffont and Tirole 1988).

[7] While complementary in production is a natural assumption in an organizational context, it is not crucial for our main result as shown in Section 6.

[8] Our results do not change if we assume a discount rate $\delta < 1$ such that $\gamma\delta > 1$.

from his individual output level q_{it} in period $t \in \{1, 2\}$. The parameter $\theta \in \{\theta^h, \theta^l\}$ represents agent i's efficiency and $1 > \theta > 0$. An agent is efficient, θ^h, with probability μ, and inefficient, θ^l, with probability $1 - \mu$, where $\Delta\theta \equiv \theta^h - \theta^l > 0$. We denote the expected efficiency of an agent by $\widetilde{\theta} \equiv \mu\theta^h + (1 - \mu)\theta^l$. The types of the two agents are drawn independently, and the prior distributions are public knowledge. Agents know their own types, while the principal does not observe the types of the agents.

In environments where agents work closely together, it is natural that an agent learns something about the other that is not observed by the top management. Moreover, when distinguishing between experienced and unexperienced workers, there exists also an asymmetry in learning abilities in that an experienced worker is more likely to learn something about the unexperienced worker than vice versa. We model these ideas in a reduced form by assuming that, before production starts in $t = 2$, agent A learns agent B's type perfectly, whereas agent B does not learn anything about agent A. This assumption is a simple illustration of the idea that agent A observes with some positive probability something about agent B's type, whereas agent B learns less about agent A.[9]

Regarding the contractual environment, we assume that the principal's long term commitment is limited in that, in $t = 1$, she cannot commit to wage transfers and output levels for $t = 2$. This captures the practical problems that prevent real life labor contracts to cover a worker's entire life span. In contrast, the principal is able to make a long term commitment to the organization's hierarchical structure. In particular, we distinguish between two organization structures denoted by $\Psi \in \{H, V\}$, which determine the contractual relationship between the principal and the two agents in $t = 2$. With the horizontal hierarchy $\Psi = H$, the principal communicates and offers contracts to both agents directly. With the vertical hierarchy $\Psi = V$, agent A becomes agent B's superior in that the principal can only communicate and interact with agent A.

Hence, our idea is that, in $t = 1$, the principal commits to the first period contract with agent A and a hierarchical structure $\Psi \in \{H, V\}$ for $t = 2$. Because agent A's type is private information, the first period contract is contingent on agent A's report on his type. Therefore, we express the first period contract Φ_1 as:

$$\Phi_1 = \{q_1(\theta_1^a),\ \omega_1(\theta_1^a)\}^{a \in \{h,l\}}, \tag{1}$$

[9] A further motivation for this asymmetry can be that the cost of information gathering is lower for the senior agent, which allows him to learn more about the junior agent.

where θ_1^a is agent A's report. At the end of $t = 1$, the contract yields the principal's and the agent's payoffs of $q_1 - \omega_1$ and $\omega_1 - (1 - \theta^a)q_1$ respectively. The timing in $t = 1$ is below:

1.1 The principal commits to $\Psi \in \{H, V\}$.

1.2 The principal offers Φ_1 to agent A.

1.3 Agent A rejects or accepts and, upon acceptance, reports his type.

1.4 The contract Φ_1 is executed.

The principal's offer in $t = 2$ depends on the organization structure $\Psi \in \{H, V\}$. Due to the complementarity in production, these offers satisfy $q_{A2} = q_{B2}$. Therefore, we only need to consider second period contracts that exhibit:

$$q \equiv \frac{q_2}{2} = q_{A2} = q_{B2} \in \{0, \gamma\}. \tag{2}$$

With $\Psi = H$, the principal offers agent A and B wages ω_A and ω_B respectively, for output level q from each agent. We denote by Φ_2^H the set of contracts for $t = 2$ that specify each agent's output level q, and the wage transfers ω_A and ω_B. For $t = 2$, the principal's payoff is $2q - \omega_A - \omega_B$, and agent A's and agent B's payoffs are $\omega_A - (1 - \theta^a)q$ and $\omega_B - (1 - \theta^b)q$ respectively. With $\Psi = H$, the timing in $t = 2$ is:

2.1 Agent A learns agent B's type.

2.2 The principal offers Φ_2^H to agents A and B.

2.3 The agents accepts/rejects. Upon acceptance, the agents send reports to the principal.

2.4 Contract Φ_2^H is executed.

With $\Psi = V$, the principal makes an offer for the total production $q_2 = 2q$ and a grand wage transfer W to agent A. In turn, agent A offers agent B a wage transfer ω_B for an output q. We denote by Φ_2^V the principal's offer to agent A for $t = 2$, which specifies q_2 and W. Agent A's offer to agent B specifies q and ω_B. For $t = 2$, the principal's payoff is $q_2 - W$, while agent A's and agent B's payoffs are $W - (1 - \theta^a)q - \omega_B$ and $\omega_B - (1 - \theta^b)q$ respectively. With $\Psi = V$, the timing in $t = 2$ is:

2.1 Agent A learns agent B's type.

2.2 The principal offers Φ_2^V to agent A.

2.3 Agent A accepts/rejects. Upon acceptance, agent A sends reports to the principal and makes an offer Φ_B to agent B.

2.4 Agent B accepts/rejects the offer from agent A.

2.5 Contracts Φ_2^V and Φ_B are executed.

Our approach is to analyze the implementable outcomes under the horizontal and the vertical hierarchy and then compare their optimality from the principal's perspective. We start our analysis with the horizontal hierarchy below.

4 Horizontal Hierarchy ($\Psi = H$)

In the horizontal hierarchy, the principal makes a second period contract offer to both agents. Using the expression in (2), the set of the second period offers, Φ_2^H, can be expressed as:

$$\Phi_2^H = \{q(\theta_A^a, \theta_A^b, \theta_B^b),\ \omega_A(\theta_A^a, \theta_A^b, \theta_B^b),\ \omega_B(\theta_A^a, \theta_A^b, \theta_B^b)\}^{a,b\in\{h,l\}}, \tag{3}$$

where θ_A^a is agent A's report on his own type, θ_A^b is agent A's report on B's type, and θ_B^b is agent B's report on his own type in $t = 2$. Note that Φ_2^H depends on the history of the game — in particular, agent A's report about his type, θ_1^a, from $t = 1$.

An advantage of the horizontal hierarchy is that the principal can directly communicate with both agents, thus can have more control. As well established in the literature, such direct communications with both agents enable the principal to elicit any shared information at no cost.[10] Because agent A and B share the information about agent B's type, the principal can elicit this information costlessly. Thus, we have the lemma below.

Lemma 1 *With $\Psi = H$, agent B's type is revealed at zero costs.*

Lemma 1 states that, in the horizontal hierarchy, only the private information about agent A matters and information revelation of agent B is not an issue. Therefore, we can

[10] One way is to require both agents to report about their shared information, and severely punish them when the reports are contradictory. Maskin and Moore (1999) and Sjöström (1999) address such schemes are even implementable with renegotiation and collusion.

treat the situation as if θ^b is publicly observable and can specify contracts as if they condition directly on agent B's true type. This allows us to express Φ_2^H as follows:

$$\Phi_2^H = \{q^{ab}, \omega_A^{ab}, \omega_B^{ab}\}^{a,b\in\{h,l\}},$$

with $q^{ab} \equiv q(\theta_A^a, \theta^b, \theta^b)$, $\omega_A^{ab} \equiv \omega_A(\theta_A^a, \theta^b, \theta^b)$, and $\omega_B^{ab} \equiv \omega_b(\theta_A^a, \theta^b, \theta^b)$.

In addition, Lemma 1 implies that agent B receives no rent. Therefore, the wage transfer to agent B in the optimal contract is:

$$\omega_B^{ab} = (1 - \theta^b)q^{ab}. \tag{4}$$

Having established these preliminary results, we turn to the issue of interim information revelation. In our dynamic framework, the degree of intermediate information revelation, determined by the principal's offer in $t = 1$, plays a crucial role. Thus, we classify the first period contract Φ_1 by the degree of information revelation it induces. For expositional reasons, here we focus on the two extreme cases where the principal induces agent A to employ a deterministic reporting strategy in $t = 1$. The principal's offer that induces no information revelation in $t = 1$ refers to the *pooling strategy*, and her offer that induces full information revelation in $t = 1$ refers to the *separating strategy.* In Appendix 2, we demonstrate that the optimality of the vertical hierarchy is robust to the consideration of semi–separating, where agent A employs a mixed reporting strategy and information revelation is only partial.

4.1 Pooling Strategy in Horizontal Hierarchy

A first period contract that does not elicit any information from agent A induces the same outcome in $t = 1$ regardless of agent A's type. Such a contract is, therefore, equivalent to a first period contract that is independent of agent A's report about his type. Hence, we express a pooling contract by a mechanism with the following structure:

$$\Phi_1^{HP} = \{q_1,\ \omega_1\}.$$

With the pooling strategy, no information is revealed in $t = 1$. Hence, in the beginning of $t = 2$, the principal still has the prior belief μ that agent A is of type θ^h. The principal's offer in $t = 2$ is a combination:

$$\Phi_2^H = \{q^{ab},\ \omega_A^{ab},\ \omega_B^{ab}\}^{a,b\in\{h,l\}},$$

where a represents agent A's report about his type in $t = 2$ and b is, by Lemma 1, agent B's actual type. Using (4), the principal's sequentially rational offer Φ_2^H maximizes her expected payoff for $t = 2$:

$$\begin{aligned}\Pi_2^H &= \mu[\mu(q^{hh} - \omega_A^{hh} + \theta^h q^{hh}) + (1-\mu)(q^{hl} - \omega_A^{hl} + \theta^l q^{hl})] \\ &\quad +(1-\mu)[\mu(q^{lh} - \omega_A^{lh} + \theta^h q^{lh}) + (1-\mu)(q^{ll} - \omega_A^{ll} + \theta^l q^{ll})],\end{aligned}$$

subject to agent A's participation constraints,

$$\omega_A^{ab} - (1-\theta^a)q^{ab} \geq 0, \quad a, b \in \{h, l\}, \tag{5}$$

and his incentive compatibility constraints,

$$\omega_A^{ab} - (1-\theta^a)q^{ab} \geq \omega_A^{a'b} - (1-\theta^a)q^{a'b}, \quad a, a', b \in \{h, l\}. \tag{6}$$

The maximization problem illustrates that the principal faces the familiar trade–off between information rent and production efficiency. Effectively, the principal has two options in $t = 2$. Her first option is to offer again a pooling contract in $t = 2$ with respect to agent A's type. Alternatively, she can offer a contract that separates the types of agent A in $t = 2$. In our setup with binary output levels, the only way to achieve separation in $t = 2$ is to abandon production depending on the agents' types. The principal's sequentially rational decision whether to separate or pool depends on her beliefs. To express this more formally, we introduce the following definition.

Definition 1 *Let* $\hat{\mu}(x) \equiv \frac{\theta^l + x}{\theta^h + x}$, *where* $x \in \mathbb{R}_+$.

With the definition above, we can characterize the sequentially rational contracts in $t = 2$, after a pooling contract in period $t = 1$ as follows.

Lemma 2 *Given* Φ_1^{HP} *in* $t = 1$, *the sequentially rational contracts* Φ_2^H *in* $t = 2$ *is:*

i) *If* $\mu \leq \hat{\mu}(\theta^l)$, *then* $q^{ab} = \gamma$; $\omega_A^{ab} = (1-\theta^l)\gamma$; $a, b \in \{h, l\}$.

ii) *If* $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$, *then* $q^{hh} = q^{hl} = q^{lh} = \gamma$, $q^{ll} = 0$; $\omega_A^{hh} = \omega_A^{lh} = (1-\theta^l)\gamma$, $\omega_A^{hl} = (1-\theta^h)\gamma$, $\omega_A^{ll} = 0$.

iii) *If* $\mu > \hat{\mu}(\theta^h)$, *then* $q^{hb} = \gamma$, $q^{lb} = 0$; $\omega_A^{hb} = (1-\theta^h)\gamma$, $\omega_A^{lb} = 0$; $b \in \{h, l\}$.

The lemma shows that the principal makes a pooling offer also for $t = 2$ when her belief about agent A's type is relatively pessimistic. This is because a more pessimistic belief shifts the trade–off between pooling and separation towards the former. According to the lemma, the principal's decision to pool or to separate in $t = 2$ depends also on agent B's type. More specifically, the principal is more eager to pool the types of agent A when agent B is efficient. The reason is that, when agent B is efficient, production is more valuable so that the principal prefers ensuring a positive output. Therefore, the disadvantage of separation that output is sometimes abandoned becomes relatively more costly to the principal.

With Lemma 2, we obtain the principal's optimal payoff from the pooling offer in $t = 1$.

Proposition 1 *With $\Psi = H$, the principal's expected payoff from the pooling strategy is:*

$$\Pi^{HP} = \begin{cases} \theta^l + (\theta^l + \widetilde{\theta})\gamma & \text{if } \mu \leq \hat{\mu}(\theta^l), \\ \theta^l + \mu(2-\mu)(\theta^h + \theta^l)\gamma & \text{if } \mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)], \\ \theta^l + \mu(\theta^h + \widetilde{\theta})\gamma & \text{if } \mu > \hat{\mu}(\theta^h). \end{cases}$$

4.2 Separating Strategy in Horizontal Hierarchy

A first period separating contract, Φ_1^{HS}, depends on agent A's report about his type and induces him to report his type truthfully. Thus, the first period separating contract has a structure as in (1) and satisfies incentive constraints that ensure agent A's truthful report.

Given a separating contract in $t = 1$, it is straightforward to derive the sequentially rational contract offer in $t = 2$. With the separating strategy, agent A's type is fully revealed with certainty in $t = 1$. This together with Lemma 1 implies that the principal makes offers in $t = 2$ as if under full information.

Lemma 3 *Given Φ_1^{HS} in $t = 1$, the sequentially rational contracts Φ_2^H in $t = 2$ is $q^{ab} = \gamma$; $\omega_A^{ab} = (1-\theta^a)\gamma$, $\omega_B^{ab} = (1-\theta^b)\gamma$, $a, b \in \{h, l\}$.*

Due to the binary production possibilities, a first period separating contract Φ^{HS} necessarily exhibits the output schedule $q_1^h = 1$ and $q_1^l = 0$. Together with Lemma 3, it follows that the following two incentive compatibility constraints must be satisfied in $t = 1$ with the separating strategy:

$$\omega_1^h - (1-\theta^h) \geq \omega_1^l + \gamma\Delta\theta, \tag{7}$$

$$\omega_1^l \geq \omega_1^h - (1-\theta^l). \tag{8}$$

Inequality (7) guarantees that the payoff of the efficient agent A is higher when he truthfully reports his type than when he misrepresents it. The LHS of (7) is agent A's total payoff over both periods if he reveals his type honestly in $t = 1$, because he receives no rent in $t = 2$. The RHS of (7) is agent A's aggregate payoff if he misrepresents his type as θ^l in $t = 1$. In such a case, the agent must produce $q_1^l = 0$ for a wage ω_1^l in $t = 1$, and expects a rent of $\omega_A^{lb} - (1 - \theta^h)\gamma = \gamma\Delta\theta$ from the sequentially rational contract in $t = 2$.

The contract Φ_1^{HS} must also satisfy (8), the incentive constraint of the inefficient type. The LHS of (8) is agent A's total payoff over the two periods when he reports his true type in $t = 1$. In the case of truth–telling, the agent produces $q_1^l = 0$ with the wage transfer ω_1^l in $t = 1$, and receives zero rent in $t = 2$. The RHS of (8) is the agent's total payoff if he misreports his type as θ^h in $t = 1$. The misreporting agent receives ω_1^h for $q_1^h = 1$ in $t = 1$, and he is offered the contract in $t = 2$ that would yield him a negative payoff, $\omega_A^{hb} - (1 - \theta^l)\gamma = -\Delta\theta\gamma$. Hence, after misreporting his type as θ^h in $t = 1$, the agent will not take the principal's offer in $t = 2$. The agent, therefore, can guarantee himself the payoff $\omega_1^h - (1 - \theta^l)$ by reporting θ^h in $t = 1$. As mentioned in the introduction, the inefficient agent achieves this payoff by employing the *take–the–money–and–run* strategy.

The following lemma shows that a horizonal hierarchy cannot achieve separation in $t = 1$, because the two incentive compatibility conditions are inconsistent.

Lemma 4 *With $\Psi = H$, the principal cannot implement a separating contract in $t = 1$.*

According to Lemma 4, information revelation in $t = 1$ is not feasible in the horizontal hierarchy. Due to the lack of commitment, the principal cannot promise agent A any rent in $t = 2$ if he revealed his true type in $t = 1$. Hence, to induce a truthful report from the efficient type in $t = 1$, the sum of rents over two periods must be paid in $t = 1$. For this reason, the principal must offer the efficient type a large wage transfer in $t = 1$. This generous first period offer is, however, also attractive to the inefficient type if he employs the take–the–money–and–run strategy. It follows from this discussion that the impossibility of an incentive compatible separating offer in $t = 1$ results from a tension between two different problems that are due to limited commitment:

1) A "hold–up" problem associated with the efficient agent.

2) A "take–the–money–and–run" problem associated with the inefficient agent.

Due to this tension, with $\Psi = H$, the principal can only make the pooling offer in $t = 1$. Thus, Proposition 1 presents the principal's maximum payoff in the horizontal hierarchy.

5 Vertical Hierarchy ($\Psi = V$)

We next analyze the structure of optimal contracts in the vertical hierarchy. Here, only agent A has a communication channel to the principal. We can therefore express the principal's second period contract with $\Psi = V$ as:

$$\Phi_2^V = \{q_2(\theta_A^a, \theta_A^b),\ W(\theta_A^a, \theta_A^b)\}^{a,b\in\{h,l\}}. \tag{9}$$

where θ_A^a and θ_A^b are agent A's reports on his own and agent B's type respectively. Due to the complementary production technology, we can in addition restrict attention to $q_2(\theta_A^a, \theta_A^b) \in \{0, 2\gamma\}$. Unlike the case with $\Psi = H$, the contract is not contingent on agent B's report θ_B^b because, with $\Psi = V$, the principal can communicate only with agent A.

If agent A accepts the offer Φ_2^V, he subsequently makes a take–it–or–leave–it offer $\{q, \omega_B\}$ to agent B. Because agent A observes agent B's type, agent B has no private information vis–à–vis agent A, and hence agent A provides no rent to agent B. In particular, agent A's offer to agent B is:

$$q = q_2/2 \text{ and } \omega_B = (1 - \theta^b)q_2/2. \tag{10}$$

With this expression it follows that agent A's payoff from the principal's second period offer q_2 and W is:

$$W - (1 - \theta^a)q_2/2 - \omega_B = W - [2 - (\theta^a + \theta^b)]q_2/2.$$

As before, we distinguish the two different extremes in first period information revelation: the pooling and the separating strategy. We first consider the case in which the principal pools the types of agent A in $t = 1$.

5.1 Pooling Strategy in Vertical Hierarchy

The structure of the pooling contract in $t = 1$ with $\Psi = V$ is the same as for $\Psi = H$:

$$\Phi_1^{VP} = \{q_1, \omega_1\}.$$

With pooling in $t = 1$, the principal does not learn agent A's private information. Because the principal cannot communicate with agent B, agent A has, from the principal's perspective, private information about both his own and agent B's type. Hence, the situation in $t = 2$ is equivalent to the principal facing an agent A with three possible types $\Theta^h \equiv 2\theta^h$,

$\Theta^m \equiv \theta^h + \theta^l$, and $\Theta^l \equiv 2\theta^l$ with the respective probabilities $\varphi^h \equiv \mu^2$, $\varphi^m \equiv 2\mu(1-\mu)$, and $\varphi^l \equiv (1-\mu)^2$. An offer $\{q_2, W\}$ yields a type Θ^j the payoff:

$$W - (2 - \Theta^j)q_2/2, \quad j \in \{h, m, l\}.$$

In $t = 2$, the sequentially rational contract Φ_2^V maximizes the principal's payoff for $t = 2$ under the agent's private information about Θ. Thus, we can express the principal's second period contract as:

$$\Phi_2^V = \{q_2^j,\ W^j\}^{j \in \{h,m,l\}}.$$

The principal's problem in $t = 2$ is then to maximize:

$$\sum_{j \in \{h,m,l\}} \varphi^j (q_2^j - W^j),$$

subject to the participation constraints,

$$W^j - (2 - \Theta^j)q_2^j/2 \geq 0, \quad j \in \{h, m, l\}, \tag{11}$$

and the incentive constraints,

$$W^j - (2 - \Theta^j)q_2^j/2 \geq W^{j'} - (2 - \Theta^j)q_2^{j'}/2, \quad j, j' \in \{h, m, l\}. \tag{12}$$

Again, the principal's sequentially rational contract in $t = 2$ treats agent A as if his efficiency can be one of the three levels. Before we present the outcome of the second period contract, we define the following notations.

Definition 2 *Let* $\underline{\mu} \equiv 1 - \sqrt{\frac{\Delta\theta}{\theta^h + \theta^l}}$ *and* $\overline{\mu} \equiv \frac{2(\theta^h + \theta^l)}{3\theta^h + \theta^l}$, *where* $\underline{\mu} < \overline{\mu}$.

With the definition above, it follows:

Lemma 5 *Given* Φ_1^{VP} *in* $t = 1$, *the sequentially rational contract* Φ_2^V *in* $t = 2$ *is:*

i) If $\mu \leq \underline{\mu}$, *then* $q_2^j = 2\gamma$; $W^j = (2 - \Theta^l)\gamma$, $j \in \{h, m, l\}$.

ii) If $\mu \in (\underline{\mu}, \overline{\mu}]$, *then* $q_2^h = q_2^m = 2\gamma$, $q_2^l = 0$; $W^h = W^m = (2 - \Theta^m)\gamma$ *and* $W^l = 0$.

iii) If $\mu > \overline{\mu}$, *then* $q_2^h = 2\gamma$, $q_2^m = q_2^l = 0$; $W^h = (2 - \Theta^h)\gamma$ *and* $W^m = W^l = 0$.

The lemma shows that when μ is small, the principal is better off pooling all three types and, therefore, provide information rents to types Θ^m and Θ^h. These information rents enable the principal to ensure production for all types. This is optimal when μ is small so that the principal is likely to face type Θ^l.

When μ is large, the principal is confident that both agent A and B are efficient. As a result, the principal chooses to separate type Θ^h from the other two, thus saving information rent completely. Although there will be no output in the case of Θ^m and Θ^l, the principal is confident enough that such cases are unlikely to be realized.

In the intermediate case, where it is likely that at least one of the agents is efficient, the principal separates Θ^l from types Θ^m and Θ^h. She thereby saves on providing an information rent to type Θ^m at the expense of forgoing production by type Θ^l. For $\mu \in (\underline{\mu}, \overline{\mu}]$, such separation is preferable because it is unlikely that the principal will face type Θ^l, but it is still likely enough that the principal will face type Θ^m.

With Lemma 5, we obtain the principal's optimal payoff from the pooling strategy in the vertical hierarchy. We show that this payoff is smaller than her payoff in the horizontal hierarchy in Proposition 1.

Proposition 2 *With $\Psi = V$, the principal's expected payoff from the pooling strategy is:*

$$\Pi^{VP} = \begin{cases} \theta^l + 2\theta^l\gamma & \text{if } \mu \leq \underline{\mu}, \\ \theta^l + [1-(1-\mu)^2]\,(\theta^h + \theta^l)\gamma & \text{if } \mu \in (\underline{\mu}, \overline{\mu}], \\ \theta^l + 2\mu^2\theta^h\gamma & \text{if } \mu > \overline{\mu}. \end{cases}$$

The principal's payoff from pooling with $\Psi = V$ is strictly smaller than the one with $\Psi = H$.

The result that the principal's payoff from the pooling strategy in the vertical hierarchy is always smaller than her optimal payoff in the horizontal hierarchy is intuitive. In the horizontal hierarchy, the principal can extract agent B's private information at no cost. In the vertical hierarchy, however, the principal cannot do so because she can communicate only with agent A. Such a restriction increases agent A's private information and makes it more costly for the principal to extract it.

The result implies that, if the principal can benefit from the vertical hierarchy at all, then the benefit must come from possibility of information revelation in $t = 1$. We turn to this case next.

5.2 Separating Strategy in Vertical Hierarchy

Similarly to the horizontal hierarchy, we can express a first period contract that fully separates the types of agent A as:

$$\Phi_1^{VS} = \{q_1^a,\ \omega_1^a\}^{a\in\{h,l\}}.$$

The contract must be incentive compatible so that agent A reports truthfully in $t = 1$. Again, due to the binary production structure, a first period separating contract exhibits $q_1^h = 1$ and $q_1^l = 0$.

With separation in $t = 1$, the principal receives an informative report θ_1^a about agent A's true type in $t = 1$. This report affects her beliefs and hence her subsequent contract in $t = 2$. Thus, we can view the principal's offer in $t = 2$ as contingent on agent A's report in $t = 1$. Because agent A learns agent B's type, and there is no direct communication between the principal and agent B, the second period contract also depends on a report of agent A about agent B's type in $t = 2$. Hence, we express the principal's second period contract as:

$$\Phi_2^V = \{q_2^{ab},\ W^{ab}\}^{a,b\in\{h,l\}},$$

where superscript a represents agent A's report about himself in $t = 1$ and superscript b represents his report about agent B in $t = 2$.

With the separating strategy, the principal is certain about agent's A type after the agent sends his report in $t = 1$. Thus, for a given θ^a with $a \in \{h, l\}$, the principal's sequentially rational contract, Φ_2^V, maximizes her payoff for $t = 2$

$$\Pi_2^V \equiv \mu(q_2^{ah} - W^{ah}) + (1-\mu)(q_2^{al} - W^{al}),$$

subject to agent A's participation constraints,

$$W^{ab} - (2 - \theta^a - \theta^b)q_2^{ab}/2 \geq 0, \quad a, b \in \{h, l\}, \tag{13}$$

and incentive constraints,

$$W^{ab} - (2 - \theta^a - \theta^b)q_2^{ab}/2 \geq W^{ab'} - (2 - \theta^a - \theta^b)q_2^{ab'}/2, \quad a, b, b' \in \{h, l\}. \tag{14}$$

Again, the principal faces a trade–off between pooling and separating — ensuring output vs. extracting information rent. In this case, however, agent A's private information in $t = 2$ is only agent B's type. Hence, the question is whether the principal wants to ensure

production for both types of agent B at the expense of providing an information rent to agent A. The alternative is to save on information rent but forego on output in the case agent B is inefficient. The next lemma shows how the principal's sequentially rational offer in $t = 2$ depends on her belief on the likelihood that agent B is efficient.

Lemma 6 *Given Φ_1^{VS}, the sequentially rational contract Φ_2^V in $t = 2$ is:*

i) If $\mu \leq \hat{\mu}(\theta^l)$, then $q_2^{ab} = 2\gamma$; $W^{ab} = (2 - \theta^a - \theta^l)\gamma$, $a, b \in \{h, l\}$; agent A gets a rent of $\Delta\theta\gamma$ in $t = 2$ when $\theta^b = \theta^h$.

ii) If $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$, then $q_2^{hh} = q_2^{hl} = q_2^{lh} = 2\gamma$, $q_2^{ll} = 0$; $W^{hh} = W^{hl} = W^{lh} = (2 - \theta^h - \theta^l)\gamma$, $W^{ll} = 0$; agent A gets a rent of $\Delta\theta\gamma$ in $t = 2$ when $\theta^a = \theta^b = \theta^h$.

iii) If $\mu > \hat{\mu}(\theta^h)$, then $q_2^{ah} = 2\gamma$, $q_2^{al} = 0$; $W^{ah} = (2 - \theta^a - \theta^h)\gamma$, $W^{al} = 0$; $a \in \{h, l\}$; agent A gets no rent in $t = 2$.

Lemma 6 follows a similar intuition as before. The trade–off between pooling and separation shifts in favor of pooling when the likelihood that agent B is efficient is smaller. The reason is that the cost of pooling — providing an information rent — is realized with a smaller probability, while the benefit — a positive output when agent B is inefficient — becomes more important for a small μ. The cutoff point of μ at which pooling prevails over separation depends on agent A's report in $t = 1$, because it determines the principal's cost of foregone revenues from abandoning output in $t = 2$.

Crucial for our subsequent analysis is agent A's rent from the sequentially rational contract, Φ_2^V, because it determines to what extent the principal can induce agent A to reveal his private information in $t = 1$. Importantly, not only agent A's rent on the equilibrium matters, but also his potential rent off the equilibrium. Therefore, we need to examine agent A's strategy for off–the–equilibrium event in which that the agent misrepresents his type in $t = 1$ and faces the sequentially rational contract Φ_2^V in $t = 2$.

Lemma 7 *The sequentially rational contract Φ_2^V induces agent A of type θ^l to reject the offer after misreporting θ^a as θ^h in $t = 1$. It induces agent A of type θ^h to accept the offer and truthfully report agent B's type after misreporting θ^a as θ^l in $t = 1$.*

As in the horizontal hierarchy, the inefficient agent A will, in case of misreporting in $t = 1$, adopt the take–the–money–and–run strategy by rejecting the principal's offer for

$t = 2$. For the efficient agent, the sequentially rational contract is individually rational and incentive compatible in that it gives the agent an incentive to report his private information about agent B honestly. Then, parallel to Lemma 4, the question is whether there exists a contract Φ_1^{VS} that keeps agent A from misreporting in $t = 1$, and induces Φ_2^V in Lemma 7.

Lemma 8 *With $\Psi = V$, Φ_1^{VS} exists when one of the following conditions is satisfied:*

i) $\mu \leq \hat{\mu}(\theta^l)$ *and* $\gamma < 1/(1-\mu)$.

ii) $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$.

iii) $\mu > \hat{\mu}(\theta^h)$ *and* $\gamma < 1/\mu$.

Lemma 8 is the counterpart of Lemma 4 and shows the paper's key insight that information revelation in $t = 1$ is easier to achieve in the vertical hierarchy than in the horizontal hierarchy. Recall that the impossibility of separation in $t = 1$ in the horizontal hierarchy was due to irreconcilable tension between the take–the–money–and–run problem associated with the *inefficient* agent and the hold–up problem associated with the *efficient* agent. In the vertical hierarchy, the principal can reconcile the tension between the two incentives as explained as follows.

First, for $\mu \leq \hat{\mu}(\theta^h)$, the sequentially rational contract Φ_2^V provides a rent to agent A regardless of his type. This rent acts as a "carrot" and alleviates the take–the–money–and–run problem, because the inefficient agent loses this future rent if he decides to run away in $t = 2$. We note the the subtlety behind this *carrot effect* — even though the rent benefits both types, it is only the rent to the *inefficient* agent that alleviates dynamic incentives.

Second, for $\mu > \hat{\mu}(\theta^l)$, the sequentially rational contract Φ_2^V, due to a loss of control, reduces production when the agent B is reported to be inefficient. Hence, the vertical hierarchy enables the principal to commit credibly to a reduced production in $t = 2$. This *play–hard–ball effect* differs from the carrot effect in that it alleviates dynamic incentives by reducing the hold–up rather than the take–the–money–and–run problem. Due to the credible production cut in $t = 2$, the *efficient* agent's information rent in that period becomes smaller, which in turn makes the agent less reluctant to reveal his information in $t = 1$.

Lemma 8 illustrates how these two strategic effects of the vertical hierarchy alleviate dynamic incentive problems. For $\mu < \hat{\mu}(\theta^l)$, the vertical hierarchy displays only the carrot effect that reduces the take–the–money–and–run problem. For $\mu > \hat{\mu}(\theta^h)$, the vertical

hierarchy displays only the play–hard–ball effect that reduces the hold–up problem. For $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h))$, the vertical hierarchy is especially effective, because it exhibits both the carrot and play–hard–ball effect, and therefore targets both the hold–up problem and the take–the–money–and–run problem.

Having addressed the implementability issue, we next derive the principal's maximum payoff from the separating strategy in the vertical hierarchy.

Proposition 3 *With $\Psi = V$, the principal's expected payoff from the separating strategy is:*

$$\Pi^{VS} = \begin{cases} \mu\theta^h + (\widetilde{\theta} + \theta^l)\gamma & \text{if } \mu \leq \hat{\mu}(\theta^l) \text{ and } \gamma < \frac{1}{1-\mu}, \\ \mu\theta^h + 2\mu(\theta^h + (1-\mu)\theta^l)\gamma & \text{if } \mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)], \\ \mu\theta^h + \mu(\theta^h + \theta^l)\gamma & \text{if } \mu > \hat{\mu}(\theta^h) \text{ and } \gamma < \frac{1}{\mu}. \end{cases}$$

To show the optimality of hierarchical structures, we introduce the following definition.

Definition 3 *Let $\widehat{\gamma} \equiv \frac{\mu\theta^h - \theta^l}{\mu^2 \Delta\theta}$.*

By Proposition 2, we only need to compare Π^{VS} above with Π^{HP} in Proposition 1. This comparison yields our main result:

Proposition 4 *The optimal hierarchical structure is characterized as follows:*

i) For $\mu \leq \hat{\mu}(0)$, the optimal hierarchy is $\Psi = H$.

ii) For $\mu \in (\hat{\mu}(0), \hat{\mu}(\theta^l)]$ and $\gamma \geq 1/(1-\mu)$, the optimal hierarchy is $\Psi = H$.

iii) For $\mu \in (\hat{\mu}(0), \hat{\mu}(\theta^l)]$ and $\gamma < 1/(1-\mu)$, the optimal hierarchy is $\Psi = V$.

iv) For $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$, the optimal hierarchy is $\Psi = V$.

v) For $\mu > \hat{\mu}(\theta^h)$ and $\gamma < \widehat{\gamma}$, the optimal hierarchy is $\Psi = V$.

vi) For $\mu > \hat{\mu}(\theta^h)$ and $\gamma \geq \widehat{\gamma}$, the optimal hierarchy is $\Psi = H$.

Figure 1 below illustrates the result. While the horizontal hierarchy provides the principal with more control, the vertical hierarchy enables her to mitigate the dynamic incentive problems that are due to limited commitment. The result shows that the principal prefers the vertical hierarchy when uncertainty about an agent's type is large. On the other hand, the principal prefers retaining direct control and extract rents in $t = 2$, when the agents are

likely to be efficient or inefficient. This demonstrates that the key trade–off in choosing a hierarchical structure is control vs. dynamic incentives.

Note also that the optimal hierarchical structure depends on the output expansion parameter γ. For example, when $\mu \in (\hat{\mu}(0), \hat{\mu}(\theta^l)]$ the vertical hierarchy prevails only if γ is not too large. The intuition is that the parameter γ measures the tension between the hold–up problem and the take–the–money–and–run strategy. The wage transfer to the efficient agent in $t = 1$ increases as γ becomes larger, and for γ large enough, the wage payment to the efficient agent makes the take–the–money–and–run profitable to the inefficient agent. As a result, separating the types of agent A in $t = 1$ becomes impossible even in the vertical hierarchy so that it has no merit over the horizontal hierarchy.

To summarize, the vertical hierarchy makes information revelation easier in $t = 1$ due to two strategic effects. First, the vertical hierarchy guarantees a future rent to the *inefficient* agent. This strategic carrot effect discourages inefficient types from playing the take–the–money–and–run strategy. Second, the vertical hierarchy allows the organization to credibly reduce the production level in the second period. This strategic play–hard–ball effect relaxes the hold–up problem that the *efficient* agent faces in $t = 1$. When the organization faces a large uncertainty about the agent's types and its growth is sluggish, the two strategic benefits dominate the loss of control and the vertical hierarchy is optimal.

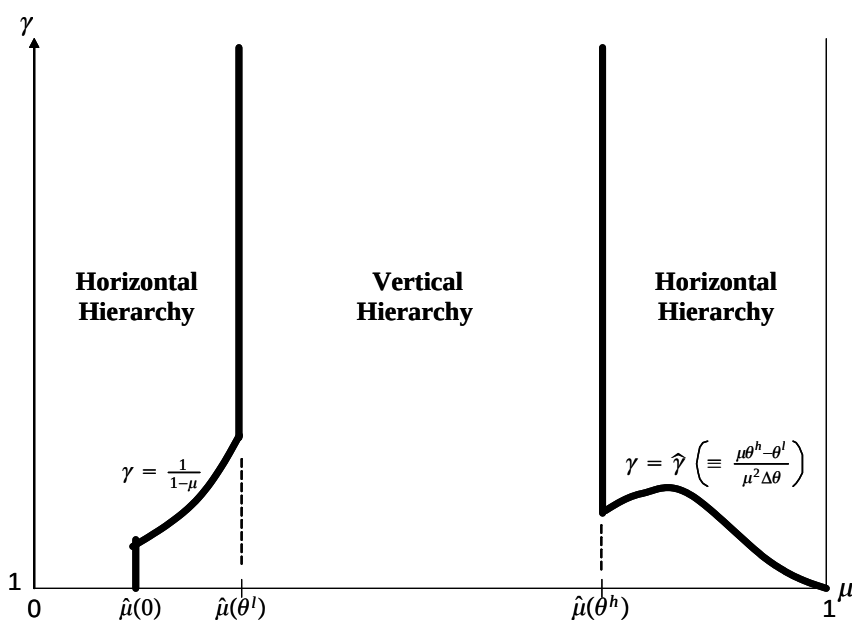

Fig 1. Horizontal vs. Vertical Hierarchy

6 Extensions

In this section, we extends our analysis to two directions. First, we look at conditional hierarchies, where the hierarchical structure in the second period depends on the senior agent's report in the first period. Second, we examine the role of complementarity in production by considering an alternative production function, with which one agent's production does not affect the other agent's production.

6.1 Conditional Hierarchy

So far we have investigated the cases in which the organization in $t = 1$ either commits to the horizontal or the vertical hierarchy. Yet, if we view the organization's long term hierarchical structure as fully contractible, then the principal could actually condition the hierarchical structure on the agent A's report (or output) in $t = 1$. In this subsection, we first show that the principal does not benefit from conditional hierarchies that promote agent A only when he is efficient. Then, we demonstrate the more counter–intuitive result that for a small μ the principal benefits from promoting agent A only when he is inefficient.

We express conditional hierarchies by the first period offer of the following structure:

$$\Phi_1^{\Psi} = \{q_1^a,\ \omega_1^a,\ \Psi^a\}^{a\in\{h,l\}},$$

with which agent A reporting type θ^a in $t = 1$ produces the output q_1^a, receives a wage ω_1^a in $t = 1$, and the hierarchical structure in $t = 2$ is $\Psi^a \in \{H, L\}$.

In principle, there are four possible combinations of which our previous analysis covered the two unconditional hierarchies $\{\Psi^h = H, \Psi^l = H\}$ and $\{\Psi^h = V, \Psi^l = V\}$. Hence, we are left to consider the two conditional combinations:

$$\Psi_{VH} \equiv \{\Psi^h = V, \Psi^l = H\} \text{ and } \Psi_{HV} \equiv \{\Psi^h = H, \Psi^l = V\}.$$

The conditional hierarchy Ψ_{VH} represents a contract that promotes agent A to middle manager only when he is efficient. By contrast, the conditional hierarchy Ψ_{HV} represents a contract that promotes the agent A only when he is inefficient.

Note that an incentive compatible conditional hierarchy necessarily implies separation so that agent A's type in $t = 1$ is fully revealing. Hence, after agent A makes a report in $t = 1$, the principal is convinced that the agent is of type θ^a. It follows that the principal's offer in

$t = 2$ depends on agent A's first period report θ_A^a as follows:

$$\Phi_2^\Psi = \begin{cases} \{q^{ab}, \omega_A^{ab}, \omega_B^{ab}\}^{b\in\{h,l\}} & \text{if } \Psi^a = H, \\ \{q_2^{ab},\ W^{ab}\}^{b\in\{h,l\}} & \text{if } \Psi^a = V. \end{cases}$$

We first study the optimality of the conditional hierarchy Ψ_{VH}. With Ψ_{VH}, the principal faces the vertical hierarchy in $t = 2$ if agent A reports θ^h in $t = 1$. In this case, the principal in $t = 2$ offers a menu $\{q_2^{hb}, W^{hb}\}^{b\in\{h,l\}}$ with the belief that agent A is efficient. The situation is, therefore, similar to the vertical hierarchy with separation, and the principal's offer for $t = 2$ coincides with the one presented in Lemma 6. If, instead, agent A reports θ^l in $t = 1$, then the principal in $t = 2$ offers a menu $\{q^{lb}, \omega_A^{lb}, \omega_B^{lb}\}^{b\in\{h,l\}}$ with the belief that agent A is inefficient. Hence, the situation is similar to the horizontal hierarchy with separation, and the principal's offer for $t = 2$ coincides with the one presented in Lemma 3. This leads to the following lemma.

Lemma 9 *The sequentially rational contract Φ_2^Ψ in $t = 2$ for a conditional hierarchy Ψ_{VH} is $q^{lh} = q^{ll} = \gamma$; $\omega_A^{lh} = \omega_A^{ll} = \omega_B^{ll} = (1-\theta^l)\gamma$, $\omega_B^{lh} = (1-\theta^h)\gamma$ and*

$$\begin{cases} q_2^{hh} = q_2^{hl} = 2\gamma;\ W^{hh} = W^{hl} = (2-\theta^h-\theta^l)\gamma & \text{if } \mu \le \widehat{\mu}(\theta^h), \\ q_2^{hh} = 2\gamma,\ q_2^{hl} = 0;\ W^{hh} = (2-\theta^h-\theta^h)\gamma,\ W^{hl} = 0 & \text{if } \mu > \widehat{\mu}(\theta^h). \end{cases}$$

Whether or not the conditional hierarchy Ψ_{VH} is actually implementable depends on the existence of wage transfers ω_1^l and ω_1^h that provide agent A with an incentive to reveal his type truthfully, when he anticipates Φ_2^Ψ as in Lemma 9. The following proposition demonstrates that this is the case, when μ and γ are small enough, but the principal's payoff with Ψ_{VH} is inferior to the principal's optimal payoff in the unconditional hierarchy $\Psi = V$.

Proposition 5 *The principal can implement Ψ_{VH} iff $\mu < \widehat{\mu}(\theta^h)$ and $\gamma < 1/(1-\mu)$. When implementable, Ψ_{VH} yields the principal a payoff $\Pi^{VH} \le \Pi^V$, where $\Pi^{VH} \equiv \mu\theta^h + (\widetilde{\theta} + \theta^l)\gamma$.*

We can interpret Ψ_{VH} as an organizational structure in which promotions are based on merit rather than seniority. Proposition 5 implies that a selective promotion of the efficient agent does not help the principal in providing incentives for information revelation. That is, with regard to promotions, seniority plays more important role than merit. The reason behind this result directly follows from the strategic merit of the vertical hierarchy — without promoting the inefficient agent, the strategic carrot effect of the vertical hierarchy disappears.

We next turn to the conditional hierarchy Ψ_{HV}. With a conditional hierarchy Ψ_{HV}, the principal faces the horizontal hierarchy in $t = 2$ if agent A reports θ^h in $t = 1$. In this case,

the principal in $t = 2$ offers a contract $\{q^{hb}, \omega_A^{hb}, \omega_B^{hb}\}^{b\in\{h,l\}}$ with the belief that agent A is efficient. Hence, the situation is similar to the horizontal hierarchy with separation, and the principal's offer for $t = 2$ coincides with the one presented in Lemma 3. If, instead, agent A reports θ^l, the principal in $t = 2$ offers a contract $\{q_2^{lb}, W^{lb}\}^{b\in\{h,l\}}$ with the belief that agent A is inefficient. The situation is similar to the vertical hierarchy with separation, and the principal's offer for $t = 2$ coincides with the one presented in Lemma 6. These considerations lead to the following result.

Lemma 10 *The sequentially rational contract Φ_2^Ψ in $t = 2$ for a conditional hierarchy Ψ_{HV} is $q^{hb} = \gamma$; $\omega_A^{hb} = (1-\theta^h)\gamma$, $\omega_B^{hb} = (1-\theta^b)\gamma$; $b \in \{h, l\}$, and*

$$\begin{cases} q_2^{lb} = 2\gamma;\ W^{lb} = (2-\theta^l-\theta^l)\gamma;\ b \in \{h,l\} & \text{if } \mu \le \widehat{\mu}(\theta^l), \\ q_2^{lh} = 2\gamma,\ q_2^{ll} = 0;\ W^{lh} = (2-\theta^l-\theta^h)\gamma,\ W^{ll} = 0 & \text{if } \mu > \widehat{\mu}(\theta^l). \end{cases}$$

As before, implementability of Ψ_{HV} depends on the existence of wage transfers ω_1^l and ω_1^h so that agent A has an incentive to reveal his type truthfully, when he anticipates Φ_2^Ψ as in Lemma 10. The following proposition shows that this is the case, when μ is large and γ is small enough.

Proposition 6 *The principal can implement Ψ_{HV} iff $\mu > \hat{\mu}(\theta^l)$ and $\gamma < 1/\mu$. When implementable, Ψ_{HV} yields the principal a payoff Π^{HV} which exceeds Π^V for $\mu > \hat{\mu}(\theta^h)$, where $\Pi^{HV} = \mu\theta^h + \mu(2-\mu)(\theta^h+\theta^l)\gamma$.*

Hence, for μ large enough, the conditional hierarchy Ψ_{HV} outperforms the unconditional vertical hierarchy.[11] To understand this result it is helpful to reconsider from the previous section, why for $\mu > \hat{\mu}(\theta^h)$, the vertical hierarchy ($\Psi = V$) performs better than the horizontal hierarchy ($\Psi = H$). Lemma 6 shows that, in the vertical hierarchy with μ large enough, production in $t = 2$ takes place only if agent B is efficient. This feature of the sequentially rational contract has a negative and a positive effect as compared to the horizontal hierarchy. The negative effect is that there is simply less production for the principal in $t = 2$. The positive effect is that the principal has to pay less rents for information revelation. As explained, it is exactly due to this smaller rent that the principal is able to induce separation in $t = 1$ and that the vertical hierarchy outperforms the horizontal one.

[11] For a study whose main focus is on promoting less efficient agent, see Fairburn and Malcomson (2001).

6.2 Role of Complementarity in Production

Until now we have shown the optimality of vertical hierarchies only for the case where agent A's and B's contributions in production are complements. In order to understand better the role of complementarity in production, we compare the vertical hierarchy to the horizontal hierarchy when production functions exhibit no complementarity.

In particular, consider the following additively separable production function for $t = 2$:

$$q_2 = q_{A2} + q_{B2}.$$

As before, each agent's individual output is q_{A2}, $q_{B2} \in \{0, \gamma\}$. The agents' outputs are now independent of each other, but the principal still benefits from employing two agents in $t = 2$.

To show a strict optimality of the vertical hierarchy with the current production function, we need a somewhat richer model. In particular, we need to drop the simplifying symmetry assumptions between agents and periods. As a result, the analysis and especially the notation becomes somewhat more tedious.

Suppose as a straightforward extension of our previous framework that agent $i \in \{A, B\}$ has productivity θ^{ih} with probability μ^i, and productivity θ^{il} with probability $1 - \mu^i$, where $\Delta\theta^i \equiv \theta^{ih} - \theta^{il} > 0$. In addition, we allow that, due to learning by doing, agent A's productivity may rise from period 1 to period 2. In particular, an agent A with productivity θ^{Aa} in $t = 1$ achieves productivity $\theta_2^{Aa} \geq \theta^{Aa}$ in period $t = 2$, where $\Delta\theta_2^A \equiv \theta_2^{Ah} - \theta_2^{Al} > 0$. Our original symmetric model obtains when $\mu^A = \mu^B$, $\theta^{Ah} = \theta_2^{Ah} = \theta^{Bh}$, and $\theta^{Al} = \theta_2^{Al} = \theta^{Bl}$.

For this richer model, we show that there exists parameter constellations for which the vertical hierarchy outperforms the horizontal hierarchy even when each agent's production contributes to the final output independently. Much of the analysis runs analog to the case with complementary production, and we explicitly point out where the analysis differs.

We first address the horizontal hierarchy ($\Psi = H$). As in the case with complementarity, with $\Psi = H$, the principal cannot implement a separating contract for agent A in $t = 1$. We therefore turn directly to the optimal pooling contract in $t = 1$. Again, agent B's productivity θ^{Bb} is revealed at no cost with $\Psi = H$ and we can treat agent B's type as observable. As before, agent B receives no rent, but contrary to the case with complementarity he always produces:

$$q_{B2} = \gamma \text{ and } \omega_B^{ab} = (1 - \theta^{Bb})\gamma, \quad a, b \in \{h, l\}.$$

Therefore, given first period pooling, the principal in $t = 2$ maximizes expected payoff:

$$\begin{aligned}\hat{\Pi}_2^H &= \mu^A\mu^B(q_{A2}^{hh} - \omega_A^{hh} + \theta^{Bh}\gamma) + \mu^A(1-\mu^B)(q_{A2}^{hl} - \omega_A^{hl} + \theta^{Bl}\gamma) \\ &\quad +(1-\mu^A)\mu^B(q_{A2}^{lh} - \omega_A^{lh} + \theta^{Bh}\gamma) + (1-\mu^A)(1-\mu^B)(q_{A2}^{ll} - \omega_A^{ll} + \theta^{Bl}\gamma),\end{aligned}$$

subject to agent A's participation and incentive compatibility conditions,

$$\omega_A^{ab} - (1-\theta_2^{Aa})q_{A2}^{ab} \geq 0, \quad a, b \in \{h, l\}, \tag{15}$$

$$\omega_A^{ab} - (1-\theta_2^{Aa})q_{A2}^{ab} \geq \omega_A^{a'b} - (1-\theta_2^{Aa})q_{A2}^{a'b}, \quad a, a', b \in \{h, l\}. \tag{16}$$

We obtain the following outcome from the sequentially rational contracts in $t = 2$, after the pooling contract Φ_1^{HP} in $t = 1$.

Lemma 11 *Given Φ_1^{HP} in $t = 1$, the sequentially rational contracts Φ_2^H in $t = 2$ is:*

i) If $\mu^A \leq \theta_2^{Al}/\theta_2^{Ah}$, then $q_{A2}^{ab} = \gamma$; $\omega_A^{ab} = (1-\theta_2^{Al})\gamma$; $a, b \in \{h, l\}$.

ii) If $\mu^A > \theta_2^{Al}/\theta_2^{Ah}$, then $q_{A2}^{hb} = \gamma$, $q_{A2}^{lb} = 0$; $\omega_A^{hb} = (1-\theta_2^{Ah})\gamma$, $\omega_A^{lb} = 0$; $b \in \{h, l\}$.

The counterpart of this lemma in the case with complementarity is Lemma 2. Hence, independent production yields only two types of sequential rational contracts rather than three. Yet, as before, the optimal first pooling contract exhibits $q_1 = 1$ and $\omega_1 = 1 - \theta^{Al}$. It then follows that the principal's optimal expected payoff for $\Psi = H$ is

$$\hat{\Pi}^{HP} = \begin{cases} \theta^{Al} + (\theta_2^l + \theta^{Bl} + \mu^B\Delta\theta^B)\gamma & \text{if } \mu \leq \theta_2^{Al}/\theta_2^{Ah}, \\ \theta_A^l + (\mu^A\theta_2^h + \theta^{Bl} + \mu^B\Delta\theta^B)\gamma & \text{if } \mu > \theta_2^{Al}/\theta_2^{Ah}. \end{cases} \tag{17}$$

We next analyze the vertical hierarchy ($\Psi = V$). Again, with pooling in $t = 1$, the vertical hierarchy is dominated by the horizontal hierarchy because the vertical hierarchy simply prevents the principal from extracting agent B's information in $t = 2$. Therefore, we turn directly to the existence and optimality of separating contract in $t = 1$. With a separating contract in $t = 1$, agent A's type is revealed at the beginning of $t = 2$ and, with an independent production function, it is always optimal for the principal to let agent A produce. Consequently,

$$q_{A2} = \gamma \text{ and } W^{ab} = (1-\theta^{Aa})\gamma + (1-\theta_A^{Bb})q_{B2}^{ab}, \quad a, b \in \{h, l\},$$

where θ_A^{Bb}, as before, denotes the agent A's report on agent B's type.

Recall that, in a vertical hierarchy with separation in $t = 1$, the principal in $t = 2$ has learned agent A's type, but is uninformed about agent B's type. Thus, for a revealed type of the agent A in $t = 1$, the principal maximizes in $t = 2$ the expected payoff:

$$\Pi_2^V = \mu^B \left(\gamma + q_{B2}^{ah} - W^{ah}\right) + (1 - \mu^B) \left(\gamma + q_{B2}^{al} - W^{al}\right),$$

subject to agent A's participation constraints,

$$W^{ab} - (1 - \theta_2^{Aa})\gamma - (1 - \theta^{Bb})q_{B2}^{ab} \geq 0, \quad b \in \{h, l\}, \tag{18}$$

and incentive constraints,

$$W^{ab} - (1 - \theta^{Bb})q_{B2}^{ab} \geq W^{ab'} - (1 - \theta^{Bb})q_{B2}^{ab'}, \quad b, b' \in \{h, l\}. \tag{19}$$

The lemma below presents the sequentially rational contracts in $t = 2$, after some separating contract Φ_1^{VS} in $t = 1$.

Lemma 12 *Given Φ_1^{VS}, the sequentially rational contract Φ_2^V in $t = 2$ is:*

i) If $\mu^B \leq \theta^{Bl}/\theta^{Bh}$, then $q_{B2}^{ab} = \gamma$; $W^{ab} = (2 - \theta_2^{Aa} - \theta^{Bl})\gamma$, $a, b \in \{h, l\}$; agent A gets a rent of $\Delta\theta^B\gamma$ in $t = 2$ when $\theta^{Bb} = \theta^{Bh}$.

ii) If $\mu^B > \theta^{Bl}/\theta^{Bh}$, then $q_{B2}^{ah} = \gamma$, $q_{B2}^{al} = 0$; $W^{ah} = (2 - \theta_2^{Aa} - \theta^{Bh})\gamma$, $W^{al} = (1 - \theta_2^{Aa})\gamma$; $a \in \{h, l\}$; agent A gets no rent in $t = 2$.

We next examine for the vertical hierarchy the incentive constraints of a separating contract in $t = 1$. The first period incentive constraint of agent A with productivity θ^{Ah} is:

$$\omega_1^h - (1 - \theta^{Ah}) + U^h(h) \geq \omega_1^l + U^h(l), \tag{20}$$

where $U^a(a')$ represents agent A's optimal continuation payoff when offered the sequentially rational contract Φ_2^V when his type is a and he reported a' in $t = 1$. Likewise, the first period incentive constraint of agent A of type θ^{Al} is:

$$\omega_1^l + U^l(l) \geq \omega_1^h - (1 - \theta^{Al}) + U^l(h). \tag{21}$$

Note that the sequentially rational contract Φ_2^V leads to a negative payoff to agent A of type θ^{Al} when he misreports his type as θ^{Ah} in $t = 1$. Hence, if this agent type intends to misreport in $t = 1$, it is best for him to follow the take–the–money–and–run strategy so that $U^l(h) = 0$. Using Lemma 12, we obtain the following result about the existence of a separating contract in $t = 1$.

Lemma 13 *With $\Psi = V$, Φ_1^{VS} exists when $\mu^B \leq \theta^{Bl}/\theta^{Bh}$ and $\gamma < \Delta\theta^A/(\Delta\theta_2^A - \mu^B\Delta\theta^B)$.*

Under the conditions of Lemma 13, separation in $t = 1$ is implementable and, analog to the complementary production case, the optimal first period wages follow from the binding participation constraint for type θ^{Al} and the binding incentive compatibility constraint for θ^{Ah} in $t = 1$:

$$\omega_1^l = -\mu^B\Delta\theta^B\gamma \text{ and } \omega_1^h = \Delta\theta_2^A\gamma - \mu^B\Delta\theta^B\gamma + (1 - \theta_2^{Ah}).$$

With these wages, the principal's optimal payoff from the first period separation is:

$$\begin{aligned}\hat{\Pi}^{VS} &= \mu^A[\theta^{Ah} - \Delta\theta_2^A\gamma + \mu^B\Delta\theta^B\gamma + (\theta_2^{Ah} + \theta^{Bl})\gamma] + (1 - \mu^A)[\mu^B\Delta\theta^B\gamma + (\theta_2^{Al} + \theta^{Bl})\gamma] \\ &= \mu^A\theta^{Ah} + (\theta_2^{Al} + \theta^{Bl})\gamma + \mu^B\Delta\theta^B\gamma. \qquad (22)\end{aligned}$$

For $\mu^A > \theta_2^{Al}/\theta_2^{Ah}$, the difference between $\hat{\Pi}^{VS}$ and $\hat{\Pi}^{HP}$ is:

$$\hat{\Pi}^{VS} - \hat{\Pi}^{HP} = [\mu^A\theta^{Ah} - \theta^{Al}] - \gamma[\mu^A\theta_2^{Ah} - \theta_2^{Al}]. \qquad (23)$$

The expression illustrates the main trade–off between the vertical and the horizontal hierarchy with an independent production function. On the RHS of (23), the first term expresses the benefit from separation in the first period, which is only possible in a vertical hierarchy. The second term expresses the benefit from separation in the second period, which occurs under the horizontal hierarchy. Hence, the vertical hierarchy is optimal exactly when separation is more beneficial in the first than in the second period.

The following proposition summarizes our discussion in this subsection.

Proposition 7 *Suppose the agents' outputs are independent of each other. The vertical hierarchy is optimal under the following conditions:*

i) $\mu^A > \theta_2^{Al}/\theta_2^{Ah}$.

ii) $\mu^B \leq \theta^{Bl}/\theta^{Bh}$.

iii) $\gamma < \Delta\theta^A/(\Delta\theta_2^A - \mu^B\Delta\theta^B)$.

iv) $\mu^A\theta^{Ah} - \theta^{Al} > \gamma(\mu^A\theta_2^{Ah} - \theta_2^{Al})$.

As shown in the previous sections, the vertical hierarchy has two strategic merits when there is complementarity among the agents' inputs. The first strategic merit is that the

vertical hierarchy discourages the inefficient agent from employing the take–the–money–and–run strategy by assuring him a rent in the second period. Proposition 7 demonstrates that under appropriate conditions this "carrot" effect also obtains without complementarity. In this sense, this first effect is robust to the absence of complementarity. While noting this robustness, we also point out that production complementarity strengthen the carrot effect, because they make it more costly for the principal to reduce the junior agent's production. Consequently, the principal is more inclined to provide the senior agent a rent for his information about his subordinate. This also explains why without complementarity, the model needs to be richer to generate the first strategic merit of the vertical hierarchy. In particular, condition *i*) and *ii*) in Proposition 7, which reflect the trade–off concerning rents, are violated in the symmetric model without complementarity.

The second strategic merit of the vertical hierarchy comes from the loss of control in the second period, which allows the organization to "play hard ball" or "use stick" more credibly in the second period. This, in turn, makes it easier for the organization to extract the agent's information in the first period. The extension reveals that such an effect vanishes in the absence of complementarity in production.

To summarize the insights of this extension, we obtain that even though complementarity is not crucial for the optimality of vertical hierarchy, it favors the vertical hierarchy for two reasons: Complementarity in production strengthens the "carrot" effect, and enables the "play-hard-ball" effect. These insights leads to the empirically testable hypothesis that vertical hierarchies are more common in organizations where the production process is more complementary.

7 Conclusion

Dynamic incentive problems arise when organizations have limited commitment. In this paper, we propose an explanation of different hierarchical structures based on such dynamic incentive problems. We show that early information revelation is easier to achieve in the vertical hierarchy, because it increases the principal's credibility to give rents and/or cut production in the future. Consequently, the optimal hierarchical structure is *vertical* when information revelation is more important, i.e., when uncertainty about the types of the agents is large. When such uncertainty is small so that information revelation is less important, the optimal hierarchy is *horizontal*. As Mintzberg (1979) points out, many professional or-

ganizations have a tendency to adopt the horizontal hierarchy. In traditional manufacturing industries, on the other hand, the variations in skill sets and efficiencies of administrative staff and line workers are often large. Our results suggest that an organization in such industries benefit from vertical hierarchies.

For simplicity, we assumed that the senior agent perfectly observes the private information of the junior agent. Our main results do not alter in a more realistic setting, where the senior agent's observation is imperfect. Crucial to our analysis is the assumption that the senior agent learns more about the junior agent than the principal. Modeling learning as a process of costly information acquisition provides a micro–foundation for such asymmetric learning abilities.[12] Indeed, it seems natural that, due to his experience, the senior agent has substantially lower costs of information acquisition than the uninvolved principal or even the inexperienced junior agent. Hence, recognizing that the senior agent's information about the junior one comes from costly information acquisition motivates the asymmetry that only the senior agent learns something about the junior agent.

Appendix 1: Proofs

Proof of Lemma 2 First note that solving the overall maximization problem follows from combining the two solutions of the submaximization problems indexed by $b \in \{h, l\}$.

$$P^b : \max_{q^{hb}, q^{lb}, \omega_A^{hb}, \omega_A^{lb}} \mu(q^{hb} - \omega_A^{hb} + \theta^b q^{hb}) + (1-\mu)(q^{lb} - \omega_A^{lb} + \theta^b q^{lb}), \text{ s.t. (5) and (6).}$$

As is standard, only the participation constraint in (5) with respect to the inefficient type θ^l and the incentive constraint in (6) with respect to the efficient type θ^h are binding. These constraints yield $\omega_A^{lb} = (1-\theta^l)q^{lb}$ and $\omega_A^{hb} = (1-\theta^h)q^{hb} + \Delta\theta q^{lb}$. After substituting these wages into the objective function, we are left to solve:

$$\max_{q^{hb}, q^{lb}} \mu(\theta^h + \theta^b)q^{hb} + (1-\mu)(\theta^l + \theta^b - [\mu/(1-\mu)]\Delta\theta)q^{lb}.$$

The objective function is increasing in q^{hb}, and hence $q^{hb} = \gamma$. Likewise, the objective function is non–decreasing in q^{lb} if $\mu \leq \hat{\mu}(\theta^b)$. Thus, $q^{lb} = \gamma$ if $\mu \leq \hat{\mu}(\theta^b)$, and $q^{lb} = 0$ otherwise. Because $\hat{\mu}(\theta^l) < \hat{\mu}(\theta^h)$ we have the three cases as specified in the lemma. ■

[12] See, for example, Lewis and Sappington (1997) or Crémer, Khalil and Rochet (1998) for agency problems with information acquisition.

Proof of Proposition 1 For $\mu \leq \hat{\mu}(\theta^l)$, it follows from Lemma 2 that the sequentially rational contract yields the principal a second period payoff of $\Pi_2^H = (\theta^l + \widetilde{\theta})\gamma$. For $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)$ this payoff is $\Pi_2^H \equiv \mu(2-\mu)(\theta^h + \theta^l)\gamma$. For $\mu > \hat{\mu}(\theta^h)$, her payoff is $\Pi_2^H = \mu(\theta^h + \widetilde{\theta})\gamma$. These payoffs are independent of the first period contract Φ_2^{HP}. The principal's optimal first period pooling contract Φ_2^{HP}, therefore, simply maximizes $\Pi_1^{HP} \equiv q_1 - \omega_1$ subject to the participation constraints of agent A over both periods. Lemma 2 shows that the inefficient agent A does not get a rent from the sequentially rational contract. His participation constraint is, therefore, $\omega_1 - (1-\theta^l)q_1 \leq 0$. Given this constraint, the contract $q_1 = 1$ and $\omega_1 = 1 - \theta^l$ maximizes Π_1^{HP} and, because this contract automatically satisfies also the participation of the efficient agent A, it is optimal. It yields the principal the payoff θ^l in $t = 1$. A simple summation of $\Pi_1^{HP} + \Pi_2^H$ yields the expression for Π^{HP}. ■

Proof of Lemma 3 From (4), $\omega_B^{ab} = (1-\theta^b)q^{ab}$. Given the report θ_1^a, the principal believes to face an agent A of type θ_1^a with certainty. Consequently, she believes that agent A's participation constraint is $\omega_A^{ab} - (1-\theta_1^a)q^{ab} \geq 0$. Maximizing the principal's profits $2q^{ab} - \omega_A^{ab} - (1-\theta^b)q^{ab}$ under the agent's participation constraint yields the result. ■

Proof of Lemma 4 Suppose separation is implementable with some first period contract with wages ω_1^h and ω_1^l. The incentive constraints (7) and (8) imply that:

$$1 - \theta^l \geq \omega_1^h - \omega_1^l \geq 1 - \theta^h + \gamma\Delta\theta.$$

To have $\omega_1^h$and ω_1^l that satisfy the inequality, $1 - \theta^l > 1 - \theta^h + \gamma\Delta\theta$ must hold, but this inequality contradicts with $\gamma > 1$. ■

Proof of Lemma 5 As usual in the model of this type, the participation constraint (11) for the most inefficient type Θ^l, and the incentive constraints (12) for the more efficient adjacent types, Θ^h (for Θ^m) and Θ^m (for Θ^l), are binding provided that this solution yields a non–decreasing schedule $q_2^h \geq q_2^m \geq q_2^l$.

The binding constraints imply, successively, $W^l = (2-\Theta^l)q_2^l/2$, $W^m = (2-\Theta^m)q_2^m/2 + (\Theta^m - \Theta^l)q_2^l/2$, and $W^h = (2-\Theta^h)q_2^h/2 + (\Theta^h - \Theta^m)q_2^m/2 + (\Theta^m - \Theta^l)q_2^l/2$. Substitution

into the objective function implies that we maximize:

$$\begin{aligned}
& \varphi^h \left[q_2^h - (2-\Theta^h)q_2^h/2 - (\Theta^h - \Theta^m)q_2^m/2 - (\Theta^m - \Theta^l)q_2^l/2\right] \\
& +\varphi^m \left[q_2^m - (2-\Theta^h)q_2^h/2 - (\Theta^m - \Theta^l)q_2^l/2\right] + \varphi^l \left[q_2^l - (2-\Theta^l)q_2^l/2\right] \\
= \; & \varphi^h \left[\Theta^h q_2^h - \Delta\theta q_2^m/2 - \Delta\theta q_2^l/2\right] + \varphi^m \left[(\Theta^h + \Theta^l)q_2^m/2 - \Delta\theta q_2^l/2\right] + \varphi^l \Theta^l q_2^l \\
= \; & q_2^h \varphi^h \Theta^h + q_2^m \left[\varphi^m(\Theta^h + \Theta^l) - \varphi^h \Delta\theta\right]/2 + q_2^l \left[\varphi^l \Theta^l - \varphi^m \Delta\theta/2 - \varphi^l \Delta\theta/2\right].
\end{aligned}$$

Maximizing the expression with respect to $q_2^h \in \{0, 2\gamma\}$ yields $q_2^h = 2\gamma$. Maximizing the expression with respect to $q_2^m \in \{0, 2\gamma\}$, we get $q_2^m = 2\gamma$ if $2\mu(1-\mu)(\theta^h + \theta^l) \geq \mu^2 \Delta\theta$, which is equivalent to $\mu \leq \bar{\mu}$, and $q_2^m = 0$ otherwise. Finally, maximizing the expression with respect to $q_2^l \in \{0, 2\gamma\}$, we get $q_2^l = 2\gamma$ if $(1-\mu)^2\theta^l \geq 2\mu(1-\mu)\Delta\theta/2 + \mu^2\Delta\theta/2$, which is equivalent to $\mu \leq \underline{\mu}$, and $q_2^l = 0$ otherwise.

The schedule q_2^h, q_2^m, q_2^l is monotone if $\overline{\mu} \geq \underline{\mu}$, because it then follows that $q_2^l = 2\gamma$ implies $q_2^m = 2\gamma$. To see that $\bar{\mu} \geq \underline{\mu}$, note that it is, per definition, equivalent to $2(\theta^h + \theta^l)/(3\theta^h + \theta^l) \geq 1 - \sqrt{\Delta\theta/(\theta^h + \theta^l)}$, which is equivalent to $\sqrt{\Delta\theta/(\theta^h + \theta^l)} \geq \Delta\theta/(3\theta^h + \theta^l)$. But the last inequality holds due to $\sqrt{\Delta\theta/(\theta^h + \theta^l)} > \Delta\theta/(\theta^h + \theta^l) > \Delta\theta/(3\theta^h + \theta^l)$. ■

Proof of Proposition 2 For $\mu \leq \underline{\mu}$, it follows from Lemma 5 that the sequentially rational contract yields the principal a second period payoff of $\Pi_2^{VP} = 2\theta^l\gamma$ and the inefficient agent A expects a payoff $\mu\Delta\theta\gamma$, because he becomes type Θ^m with probability μ. For $\mu \in (\underline{\mu}, \overline{\mu}]$ this payoff is $\Pi_2^{VP} = [1 - (1-\mu)^2](\theta^h + \theta^l)\gamma$ and the inefficient agent A expects a zero payoff. For $\mu > \overline{\mu}$, her payoff is $\Pi_2^{VP} = 2\mu^2\theta^h\gamma$ and the inefficient agent A expects a zero payoff. These payoffs are independent of the first period contract Φ_2^V. The principal's optimal first period pooling contract Φ_2^V, therefore, simply maximizes $\Pi_1^{VP} \equiv q_1 - \omega_1$ subject to the participation constraints of agent A over both periods. The inefficient agent A receive a rent in $t = 2$ only for $\mu \leq \underline{\mu}$. In this case, his participation constraint is $\omega_1 - (1-\theta^l)q_1 + \mu\Delta\theta\gamma \leq 0$ so that the optimal first period contract is $q_1 = 1$ and $\omega_1 = (1-\theta^l)q_1 - \mu\Delta\theta\gamma \leq 0$. It yields the principal the payoff $\theta^l + \mu\Delta\theta\gamma$ in $t = 1$. For $\mu > \underline{\mu}$, the inefficient agent A does not expect a rent from the sequentially rational contract so that his participation constraint is $\omega_1 - (1-\theta^l)q_1 \leq 0$. Hence, the optimal first period contract is $q_1 = 1$ and $\omega_1 = 1 - \theta^l$. It yields the principal the payoff θ^l in $t = 1$. Adding the payoffs over both period yields the expression for Π^{VP} in the Proposition.

Next, we show that $\Delta\Pi^P \equiv \Pi^H - \Pi^{VP} \geq 0$ for all $\mu \in [0, 1]$. In order to compute $\Delta\Pi^P$

we first claim that $\underline{\mu} < \hat{\mu}(\theta^l)$ and $\overline{\mu} > \hat{\mu}(\theta^h)$. The first claim follows because

$$\hat{\mu}(\theta^l) - \underline{\mu} = \frac{2\theta^l}{\theta^h + \theta^l} - 1 + \sqrt{\frac{\Delta\theta}{\theta^h + \theta^l}} = -\frac{\Delta\theta}{\theta^h + \theta^l} + \sqrt{\frac{\Delta\theta}{\theta^h + \theta^l}},$$

which is positive because the fraction is smaller than 1. The second claim follows from

$$\overline{\mu} - \hat{\mu}(\theta^h) = \frac{2(\theta^h + \theta^l)}{3\theta^h + \theta^l} - \frac{\theta^l + \theta^h}{2\theta^h} = \frac{{\theta^h}^2 - {\theta^l}^2}{2\theta^h(3\theta^h + \theta^l)} > 0.$$

Consequently,

$$\Delta\Pi^P = \begin{cases} 0 & \text{if } \mu \leq \underline{\mu} \\ (1-\mu)(2\theta^l - \mu(\theta^h + \theta^l))\gamma & \text{if } \underline{\mu} < \mu \leq \hat{\mu}(\theta^l) \\ 0 & \text{if } \hat{\mu}(\theta^l) < \mu \leq \hat{\mu}(\theta^h) \\ \mu(2\mu\theta^h - (\theta^h - \theta^l))\gamma & \text{if } \hat{\mu}(\theta^h) < \mu \leq \overline{\mu} \\ (1-\mu)\mu(\theta^h + \theta^l)\gamma & \text{if } \mu > \overline{\mu}. \end{cases}$$

These terms are all non–negative, because $\mu \leq \hat{\mu}(\theta^l)$ implies $2\theta^l \geq \mu(\theta^h + \theta^l)$ and $\mu > \hat{\mu}(\theta^h)$ implies $2\mu\theta^h > \theta^h - \theta^l$. ■

Proof of Lemma 6 Given the report θ_1^a in $t = 1$, the maximization problem is equivalent to a static contracting problem where there is private information about type $\theta^b \in \{\theta^h, \theta^l\}$. As in Lemma 2, the participation of the inefficient type and the incentive compatibility constraint of the efficient type are binding. This yields wages:

$$W^{al} = (2 - \theta^a - \theta^l)q_2^{al}/2 \text{ and } W^{ah} = (2 - \theta^a - \theta^h)q_2^{ah}/2 + \Delta\theta q_2^{al}/2.$$

Substituting these variables in the principal's objective function yields:

$$\begin{aligned} & \mu[q_2^{ah} - (2 - \theta^a - \theta^h)q_2^{ah}/2 - \Delta\theta q_2^{al}/2] + (1-\mu)[q_2^{al} - (2 - \theta^a - \theta^l)q_2^{al}/2] \\ = \; & \mu q_2^{ah}[\theta^a + \theta^h]/2 + q_2^{al}[(1-\mu)(\theta^a + \theta^l) - \mu\Delta\theta]/2. \end{aligned}$$

Maximizing this expression for $q_2^{ah} \in \{0, 2\gamma\}$ yields $q_2^{ah} = 2\gamma$. Maximizing the expression for $q_2^{al} \in \{0, 2\gamma\}$ yields $q_2^{al} = 2\gamma$ if $(1-\mu)(\theta^a + \theta^l) \geq \mu\Delta\theta$, which is equivalent to $\mu \leq \hat{\mu}(\theta^a)$, and $q_2^{al} = 0$ otherwise. Because $\hat{\mu}(\theta^h) > \hat{\mu}(\theta^l)$ the lemma follows. ■

Proof of Lemma 7 After reporting $\theta^a = \theta^h$ in $t = 1$, the inefficient agent A receives, as specified in Lemma 6, the contract $\{q_2^{hb}, W^{hb}\}^{b\in\{h,l\}}$ when he reports agent B's type in $t = 2$. That is, he receives the payoff $W^{lb} - (1-\theta^l)q_2^{lb}/2 - (1-\theta^{b'})q_2^{lb}/2$ from accepting the contract when agent B is actually of type $\theta^{b'}$ (which may differ from the reported θ^b). It is straightforward to check that, for the contracts in Lemma 6 these payoffs are all non–positive for all combinations (b, b') and any $\mu \in [0, 1]$ so that it is optimal for the inefficient agent to reject the contract.

After reporting $\theta^a = \theta^l$ in $t = 1$, the efficient agent A receives the contract $\{q_2^{lb}, W^{lb}\}^{b\in\{h,l\}}$ as specified in Lemma 6. That is, he receives the payoff $W^{lb} - (1-\theta^h)q_2^{lb}/2 - (1-\theta^{b'})q_2^{lb}/2$ from accepting the contract when agent B is actually of type $\theta^{b'}$. It is straightforward to check that, for the contracts in Lemma 6, the efficient agent A always has a weakly higher payoff from reporting agent B's type $\theta^{b'}$ honestly and this payoff is non–negative. ■

Proof of Lemma 8 We distinguish the three different cases of Lemma 6.

Case i) $\mu \leq \hat{\mu}(\theta^l)$: From Lemma 6 it follows that the efficient agent A receives an expected information rent $\mu\Delta\gamma$ if he reports θ^h in $t = 1$. If instead he reports θ^l then, by Lemma 7 and 6, he receives a rent in $t = 2$ of $2\Delta\theta\gamma$ if agent B happens to be efficient and $\Delta\theta\gamma$ if agent B happens to be inefficient. Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^h if:

$$\omega_1^h - (1-\theta^h) + \mu\Delta\theta\gamma \geq \omega_1^l + \Delta\theta\gamma + \mu\Delta\theta\gamma.$$

Likewise, it follows from Lemma 6 and Lemma 7 that, irrespective of his report in $t = 1$, the inefficient agent A receives no rent. Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^l if:

$$\omega_1^l + \mu\Delta\theta\gamma \geq \omega_1^h - (1-\theta^l).$$

Combining the previous two conditions shows that the contract Φ_1^{VS} is incentive compatible exactly when:

$$(1-\theta^h) + \Delta\theta\gamma \leq \omega_1^h - \omega_1^l \leq (1-\theta^l) + \mu\Delta\theta\gamma. \tag{24}$$

Implementability requires $1 - \theta^h + \Delta\theta\gamma < 1 - \theta^l + \mu\Delta\theta\gamma$ which holds exactly when $\gamma < 1/(1-\mu)$.

Case ii) $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$: From Lemma 6 and Lemma 7 it follows that the efficient agent A receives an expected information rent of $\mu\Delta\theta\gamma$ irrespective of his report in $t = 1$.

Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^h if:

$$\omega_1^h - (1-\theta^h) + \mu\Delta\theta\gamma \geq \omega_1^l + \mu\Delta\theta\gamma.$$

Likewise, it follows from Lemma 6 that the inefficient agent A receives an expected rent $\mu\Delta\theta\gamma$ in $t=2$ from reporting θ^l. If he reports θ^h instead then, by Lemma 7, he receives no rent in $t=2$. Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^l if:

$$\omega_1^l + \mu\Delta\theta\gamma \geq \omega_1^h - (1-\theta^l).$$

Combining the previous two conditions shows that the contract Φ_1^{VS} is incentive compatible exactly when:

$$(1-\theta^h) \leq \omega_1^h - \omega_1^l \leq (1-\theta^l). \tag{25}$$

Hence, implementability requires $1-\theta^h < 1-\theta^l$ which is always the case.

Case iii) $\mu > \hat{\mu}(\theta^h)$: From Lemma 6 and Lemma 7 it follows that the efficient agent A receives no rent in $t=2$ if he reports θ^h in $t=1$. If, instead, he reports θ^l then Lemma 7 and 6 imply that he receives a rent in $t=2$ of $\Delta\theta\gamma$ in $t=2$ if agent B happens to be efficient and no rent if agent B happens to be inefficient. Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^h if:

$$\omega_1^h - (1-\theta^h) \geq \omega_1^l + \mu\Delta\theta\gamma.$$

Likewise, it follows from Lemma 6 that, irrespective of his report in $t=2$, the inefficient agent A receives no rent in $t=2$. Hence, the contract Φ_1^{VS} is incentive compatibility to type θ^l if:

$$\omega_1^l \geq \omega_1^h - (1-\theta^l).$$

Combining the previous two conditions shows that the contract Φ_1^{VS} is incentive compatible exactly when:

$$(1-\theta^h) + \mu\Delta\theta\gamma \leq \omega_1^h - \omega_1^l \leq (1-\theta^l). \tag{26}$$

Implementability requires $1-\theta^h + \mu\Delta\theta\gamma < 1-\theta^l$ which holds exactly when $\gamma < 1/\mu$. ■

Proof of Proposition 3 The principal's optimal wage structure minimizes her expected wages:

$$E_\theta\left[\omega_1^a\right] = \mu\omega_1^h + (1-\mu)\omega_1^l,$$

and we are left to determine ω_1^h and ω_1^l. For $\mu \leq \hat{\mu}(\theta^l)$, the principal minimizes $E_\theta[\omega_1^a]$ under the incentive constraints (24) and the participation constraints,

$$\omega_1^h - (1-\theta^h) + \mu\Delta\theta\gamma \geq 0 \text{ and } \omega_1^l + \mu\Delta\theta\gamma \geq 0.$$

At the optimum the participation constraint of type θ^l and the incentive constraint of type θ^h are binding. This yields $\omega_1^h = 1 - \theta^h + (1-\mu)\Delta\theta\gamma$ and $\omega_1^l = -\mu\Delta\theta\gamma$. The principal's payoff is $\Pi^V = \mu\theta^h + (\tilde{\theta} + \theta^l)\gamma$.

For $\hat{\mu}(\theta^l) \leq \mu \leq \hat{\mu}(\theta^h)$, the principal minimizes $E_\theta[\omega_1^a]$ under the incentive constraints (25) and the participation constraints,

$$\omega_1^h - (1-\theta^h) + \mu\Delta\theta\gamma \geq 0 \text{ and } \omega_1^l \geq 0.$$

At the optimum the participation constraint of type θ^l and θ^h are binding. This yields $\omega_1^h = 1 - \theta^h - \mu\Delta\theta\gamma$ and $\omega_1^l = 0$. The principal's payoff is $\Pi^V = \mu(\theta^h + \mu\Delta\theta\gamma) + (1-(1-\mu)^2)(\theta^h + \theta^l)\gamma = \mu\theta^h + 2\mu(\theta^h + (1-\mu)\theta^l)\gamma$.

For $\mu > \hat{\mu}(\theta^h)$ we minimize $E_\theta[\omega_1^a]$ under the incentive constraints (26) and the participation constraints,

$$\omega_1^h - (1-\theta^h) \geq 0 \text{ and } \omega_1^l \geq 0.$$

At the optimum the participation constraint of type θ^l and the incentive constraint of type θ^h are binding. This yields $\omega_1^h = 1 - \theta^h + \mu\Delta\theta\gamma$ and $\omega_1^l = 0$. The principal's payoff is $\Pi^V = \mu(\theta^h - \mu\Delta\theta\gamma) + [2\mu^2\theta^h + (1-\mu)\mu(\theta^h + \theta^l)\gamma = \mu\theta^h + \mu(\theta^h + \theta^l)\gamma$. ■

Proof of Proposition 4 By Proposition 2, the horizontal hierarchy is optimal whenever first period separation in the vertical hierarchy is not implementable. We therefore only need to compare Π^H with Π^V. From Proposition 1 and 3, we compute $\Delta\Pi = \Pi^H - \Pi^V$ whenever first period separation in the vertical hierarchy is implementable. For $\mu \leq \hat{\mu}(\theta^l)$ and $\gamma < 1/(1-\mu)$ it follows $\Delta\Pi = \theta^l - \mu\theta^h$, which is positive exactly when $\mu < \theta^l/\theta^h = \hat{\mu}(0)$. Hence, if $\mu < \hat{\mu}(0)$ the horizontal hierarchy is optimal irrespective of whether γ. Instead, for $\hat{\mu}(0) < \mu \leq \hat{\mu}(\theta^l)$, the vertical hierarchy is optimal whenever it is implementable which is the case for $\gamma \geq 1/(1-\mu)$. For $\hat{\mu}(\theta^l) < \mu \leq \hat{\mu}(\theta^h)$, we have $\Delta\Pi = \theta^l - \mu\theta^h - \mu^2\Delta\theta < -\mu^2\Delta\theta < 0$, where the first inequality follows because $\mu \geq \hat{\mu}(\theta^l) > \hat{\mu}(0)$. Hence, the vertical hierarchy is optimal whenever implementable and implementability is always ensured. For $\mu > \hat{\mu}(\theta^h)$ and $\gamma < 1/\mu$ we have $\Delta\Pi = \theta^l - \mu\theta^h + \mu(\tilde{\theta} - \theta^l)\gamma = \theta^l - \mu\theta^h + \mu^2\Delta\theta\gamma$. This is negative exactly when $\gamma < \mu(\theta^h - \theta^l)/(\mu^2\Delta\theta)$. Because $\mu(\theta^h - \theta^l)/(\mu^2\Delta\theta) < \mu(\theta^h - \mu\theta^l)/(\mu^2\Delta\theta) < 1/\mu$. $\Delta\Pi < 0$ implies $\gamma < 1/\mu$ so that first period separation is implementable. ■

Proof of Lemma 9 With Ψ_{VH}, the sequentially rational contract $\{q_2^{ab},\ W^{ab}\}^{b\in\{h,l\}}$ after a first period report θ^h maximizes Π_2^V subject to (13) and (14) with $\theta^a = \theta^h$. Lemma 6 provides the solution to this problem. With Ψ_{VH}, the sequentially rational contract $\{q^{ab}, \omega_A^{ab}, \omega_B^{ab}\}^{b\in\{h,l\}}$ in $t = 2$ after agent A's report that $\theta^a = \theta^l$ in $t = 1$ follows directly from Lemma 3. ■

Proof of Proposition 5 From the first period contract Φ_1^Ψ with Ψ_{VH}, agent A of type θ^l expects the payoff:

$$\omega_1^l - (1-\theta^l)q_1^l + (1-\theta^l)\gamma - (1-\theta^l)\gamma = \omega_1^l - (1-\theta^l)q_1^l \tag{27}$$

from reporting type θ^l in $t = 1$. If this type, in contrast, reports θ^h, then it is optimal for him to reject the sequentially rational contract offer Φ_2^Ψ of Lemma 9 in $t = 2$ for both parameter constellations $\mu \leq \hat{\mu}(\theta^h)$ and $\mu > \hat{\mu}(\theta^h)$. Hence, reporting θ^h in $t = 2$ yields type θ^l the payoff:

$$\omega_1^h - (1-\theta^l)q_1^h. \tag{28}$$

Type θ^l has an incentive to report truthfully in $t = 1$ exactly when (27) exceeds (28), i.e., exactly when

$$\omega_1^h - \omega_1^l \leq (1-\theta^l)(q_1^h - q_1^l). \tag{29}$$

For the parameter constellation $\mu \leq \hat{\mu}(\theta^h)$, it follows from Lemma 9 that truthfully reporting his type θ^h yields agent A:

$$\omega_1^h-(1-\theta^h)q_1^h+(2-\theta^h-\theta^l)\gamma-(1-\theta^h)\gamma-\mu(1-\theta^h)\gamma-(1-\mu)(1-\theta^l)\gamma = \omega_1^h-(1-\theta^h)q_1^h+\mu\Delta\theta\gamma. \tag{30}$$

For the parameter constellation $\mu > \hat{\mu}(\theta^h)$, it follows from Lemma 9 that truthfully reporting his type θ^h yields agent A:

$$\omega_1^h - (1-\theta^h)q_1^h + \mu[(2-\theta^h-\theta^h)\gamma - (1-\theta^h)\gamma - (1-\theta^h)\gamma] = \omega_1^h - (1-\theta^h)q_1^h. \tag{31}$$

Instead, reporting θ^l in $t = 1$ yields agent A of type θ^h:

$$w_1^l - (1-\theta^h)q_1^l + (1-\theta^l)\gamma - (1-\theta^h)\gamma = w_1^l - (1-\theta^h)q_1^l + \Delta\theta\gamma. \tag{32}$$

Hence, for $\mu \leq \hat{\mu}(\theta^h)$ agent A of type θ^h has an incentive to report honestly exactly when (30) exceeds (32), i.e., exactly when

$$\omega_1^h - \omega_1^l \geq (1-\theta^h)(q_1^h - q_1^l) + (1-\mu)\Delta\theta\gamma. \tag{33}$$

This is consistent with (29) only if $q_1^h = 1$, $q_1^l = 0$ and $(1-\mu)\gamma \leq 1$. Similarly, for $\mu > \hat{\mu}(\theta^h)$ agent A of type θ^h has an incentive to report honestly exactly when (31) exceeds (32), i.e., when

$$\omega_1^h - \omega_1^l \geq (1-\theta^h)(q_1^h - q_1^l) + \Delta\theta\gamma. \tag{34}$$

This is however always inconsistent with (29).

This proves the first claim of the proposition that only for $\mu \leq \hat{\mu}(\theta^h)$ and $(1-\mu)\gamma \leq 1$, the conditional hierarchy is implementable. Moreover, implementability requires $q_1^h = 1$ and $q_1^l = 0$ so that the principal's payoff is:

$$\mu[1-\omega_1^h+2\gamma-(2-\theta^h-\theta^l)\gamma]+(1-\mu)[-w_1^l+2\gamma-(1-\theta^l)\gamma-\mu(1-\theta^h)\gamma-(1-\mu)(1-\theta^l)\gamma] \tag{35}$$

The optimal contract maximizes (35) subject to $q_1^h = 1$, $q_1^l = 0$, the incentive constraints (29) and (33), and the participation constraints which require that (27) and (31) are non–negative. Straightforward calculations yield that this problem is maximized for $\omega_1^h = 1-\theta^h+(1-\mu)\Delta\theta\gamma$ and $\omega_1^l = 0$ with payoff:

$$\Pi^{VH} = \mu\theta^h + (\tilde{\theta} + \theta^l). \tag{36}$$

From Proposition 3 and 4 we can compare this expression with Π^V for the implementable range $\mu \leq \hat{\mu}(\theta^h)$ and $(1-\mu)\gamma \leq 1$. This yields $\Pi^{VH} = \Pi^V$ when $\mu \in (\hat{\mu}(0), \hat{\mu}(\theta^l)]$ and $\Pi^{VH} - \Pi^V = [(2-\mu)\theta^l - 2\mu(1-\mu)\theta^l - \mu\theta^h]\gamma < 0$ for $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$ and $\gamma < 1/(1-\mu)$. This proves the second claim of the proposition. ■

Proof of Lemma 10 With Ψ_{HV}, the sequentially rational contract $\{q_2^{ab},\ W^{ab}\}^{b\in\{h,l\}}$ after a first period report θ^l maximizes Π_2^V subject to (13) and (14) with $\theta^a = \theta^l$. Lemma 6 provides the solution to this problem. With Ψ_{HV}, the sequentially rational contract $\{q^{ab}, \omega_A^{ab}, \omega_B^{ab}\}^{b\in\{h,l\}}$ in $t = 2$ after agent A's report that $\theta^a = \theta^h$ in $t = 1$ follows directly from Lemma 3. ■

Proof of Proposition 6 First, consider the parameter constellation $\mu \leq \hat{\mu}(\theta^l)$. From the first period contract Φ_1^Ψ with Ψ_{HV}, agent A of type θ^l expects the payoff:

$$\omega_1^l-(1-\theta^l)q_1^l+2(1-\theta^l)\gamma-(1-\theta^l)\gamma-\mu(1-\theta^h)\gamma-(1-\mu)(1-\theta^l)\gamma = \omega_1^l-(1-\theta^l)q_1^l+\mu\Delta\theta\gamma \tag{37}$$

from reporting type θ^l in $t = 1$. If this type, in contrast, reports θ^h, then it is optimal for him to reject the sequentially rational contract offer Φ_2^Ψ of Lemma 9 in $t = 2$. Hence, reporting

θ^h in $t = 2$ yields type θ^l the payoff:

$$\omega_1^h - (1 - \theta^l)q_1^h. \tag{38}$$

Hence, type θ^l has an incentive to report truthfully in $t = 1$ exactly when (37) exceeds (38), i.e., exactly when

$$\omega_1^h - \omega_1^l \leq (1 - \theta^l)(q_1^h - q_1^l) + \mu\Delta\theta\gamma. \tag{39}$$

From Lemma 10 it follows that truthfully reporting his type θ^h yields agent A:

$$\omega_1^h - (1 - \theta^h)q_1^h + (1 - \theta^h)\gamma - (1 - \theta^h)\gamma = \omega_1^h - (1 - \theta^h)q_1^h. \tag{40}$$

Instead, reporting θ^l in $t = 1$ yields agent A of type θ^h:

$$\omega_1^l - (1-\theta^h)q_1^l + (2-\theta^l-\theta^l)\gamma - (1-\theta^h)\gamma - \mu(1-\theta^h)\gamma - (1-\mu)(1-\theta^l)\gamma = w_1^l - (1-\theta^h)q_1^l + (1+\mu)\Delta\theta\gamma. \tag{41}$$

Type θ^h has an incentive to report truthfully when (40) exceeds (41), i.e., whenever

$$\omega_1^h - \omega_1^l \geq (1 - \theta^h)(q_1^h - q_1^l) + (1 + \mu)\Delta\theta\gamma. \tag{42}$$

Because $\gamma > 1 \geq q_1^h - q_1^l$, this incentive constraint is never consistent with (39).

Next, consider the parameter constellation $\mu > \hat{\mu}(\theta^l)$. In this case, truthfully reporting his type θ^l yields agent A the payoff:

$$\omega_1^l - (1 - \theta^l)q_1^l + \mu[(2 - \theta^l - \theta^h)\gamma - (1 - \theta^l)\gamma - (1 - \theta^h)\gamma = \omega_1^l - (1 - \theta^l)q_1^l. \tag{43}$$

Also for $\mu > \hat{\mu}(\theta^l)$, this type rejects the sequentially rational contract offer Φ_2^Ψ after reporting θ^h instead. Hence, reporting θ^h in $t = 2$ yields type θ^l the payoff (38). Hence, type θ^l has an incentive to report truthfully in $t = 1$ exactly when (43) exceeds (38), i.e., exactly when

$$\omega_1^h - \omega_1^l \leq (1 - \theta^l)(q_1^h - q_1^l). \tag{44}$$

From Lemma 10 it follows that truthfully reporting his type θ^h yields agent A the payoff (40). Instead, reporting θ^l in $t = 1$ yields agent A of type θ^h the following payoff[13]:

$$w_1^l - (1 - \theta^h)q_1^l + \mu[(2 - \theta^l - \theta^h)\gamma - (1 - \theta^h)\gamma - (1 - \theta^h)\gamma] = w_1^l - (1 - \theta^h)q_1^l + \mu\Delta\theta\gamma. \tag{45}$$

[13] As before, the sequential rational contract in Lemma 10 that is offered after a first period report θ^l is also incentive compatible for agent A of type θ^h.

Hence, type θ^h has an incentive to report truthfully when (40) exceeds (45), i.e., whenever

$$\omega_1^h - \omega_1^l \geq (1-\theta^h)(q_1^h - q_1^l) + \mu\Delta\theta\gamma. \tag{46}$$

This incentive constraint is only consistent with (44) if $q_1^h = 1$, $q_1^l = 0$, and $\mu\gamma \leq 1$. This proves the first statement of the proposition.

Hence, implementability requires $\mu > \hat{\mu}(\theta^l)$, $q_1^h = 1$, $q_1^l = 0$, and $\mu\gamma \leq 1$ so that the principal's payoff is:

$$\mu[1-\omega_1^h+2\gamma-(1-\theta^h)\gamma-\mu(1-\theta^h)\gamma-(1-\mu)(1-\theta^l)\gamma]+(1-\mu)[-w_1^l+2\gamma-(2-\theta^l-\theta^h)\gamma]$$

$$= \mu[1-\omega_1^h+(\theta^h+\tilde{\theta})\gamma]+(1-\mu)[-\omega_1^l+\mu(\theta^l+\theta^h)\gamma] \tag{47}$$

The optimal contract maximizes (47) subject to $q_1^h = 1$, $q_1^l = 0$, the incentive constraints (44) and (46), and the participation constraints which require that (40) and (43) are non–negative. Straightforward calculations yield that this problem is maximized for $\omega_1^l = 0$ and $\omega_1^h = 1 - \theta^h + \mu\Delta\theta\gamma$ with the payoff:

$$\Pi^{HV} = \mu\theta^h + \mu(2-\mu)(\theta^h+\theta^l)\gamma. \tag{48}$$

Using Proposition 3 and 4, we can compare (48) with (i) Π^V for $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$ and $\gamma < 1/\mu$, and (ii) Π^V for $\mu > \hat{\mu}(\theta^h)$ and $\gamma < \frac{\mu\theta^h-\theta^l}{\mu^2\Delta\theta}$. From a direct comparison, $\Pi^{HV} - \Pi^V = -\mu^2\Delta\theta\gamma < 0$ in case (i) and $\Pi^{HV} - \Pi^V = \mu(1-\mu)\Delta\theta\gamma > 0$ in case (ii). ■

Proof of Lemma 11 As before, the two participation constraints in (15) with $a = l$ and $b \in \{h, l\}$ and the two incentive constraint in (16) with $a = l$, $a' = h$, and $b \in \{h, l\}$ are binding. These constraints yield $\omega_A^{lb} = (1-\theta_2^{Al})q_{A2}^{lb}$ and $\omega_A^{hb} = (1-\theta_2^{Ah})q_{A2}^{hb} + \Delta\theta_2^A q_{A2}^{lb}$. Because agent B's type is observable at zero costs, we can break up the principal's overall maximization problem into two independent ones. In particular, the principal maximizes for each $b \in \{h, l\}$

$$\max_{q_{A2}^{hb}, q_{A2}^{lb}} \mu^A(\theta_2^{Ah}q_{A2}^{hb} - \Delta\theta_2^A q_{A2}^{lb}) + (1-\mu^A)\theta_2^{Al}q_{A2}^{lb} + \theta^{Bb}\gamma.$$

The objective function is increasing in q_{A2}^{hb} and thus $q_{A2}^{hb} = \gamma$. Likewise, the objective function is non-decreasing in q_{A2}^{lb} if $\mu^A \leq \theta_2^{Al}/\theta_2^{Ah}$. Thus, $q_{A2}^{lb} = \gamma$ if $\mu^A \leq \theta_2^l/\theta_2^h$, and $q_{A2}^{lb} = 0$ otherwise. ■

Proof of Lemma 12 With the constraint (18) only binding for $b = l$ and (19) only binding for $b = h$ and $b' = l$, it follows $W^{al} = (1 - \theta_2^{Aa})\gamma + (1 - \theta^{Bl})q_{B2}^{al}$ and $W^{ah} = (1-\theta_2^{Aa})\gamma + (1-\theta^{Bh})q_{B2}^{ah} + \Delta\theta^B q_{B2}^{al}$. With these expressions, the principal's problem in $t = 2$ is written as:

$$\max_{q_{B2}^{ah}, q_{B2}^{al}} \mu^B \left(\theta^{Bh} q_{B2}^{ah} - \Delta\theta^B q_{B2}^{al}\right) + (1 - \mu^B)\theta^{Bl} q_{B2}^{al} + \theta_2^{Aa}\gamma.$$

The objective function is increasing in q_{B2}^{ah}, and thus $q_{B2}^{ah} = \gamma$. Since the objective function is non-decreasing in q_{B2}^{al} if $\mu^B \leq \theta^{Bl}/\theta^{Bh}$, $q_{B2}^{al} = \gamma$ if $\mu^B \leq \theta^{Bl}/\theta^{Bh}$. Otherwise, $q_{B2}^{al} = 0$. ■

Proof of Lemma 13 Following Lemma 12 we distinguish between two cases:

Case i) $\mu^B \leq \theta^{Bl}/\theta^{Bh}$: From Lemma 12, it follows $U^h(h) = U^l(l) = \mu^B\Delta\theta^B\gamma$ and $U^h(l) = \Delta\theta_2^A\gamma + \mu^B\Delta\theta^B\gamma$. Incentive constraint (20) therefore becomes:

$$\omega_1^h - (1 - \theta^{Ah}) + \mu^B\Delta\theta^B\gamma \geq \omega_1^l + \Delta\theta_2^A\gamma + \mu^B\Delta\theta^B\gamma.$$

Because in addition $U^l(h) = 0$, incentive constraint (21) becomes:

$$\omega_1^l + \mu^B\Delta\theta^B\gamma \geq \omega_1^h - (1 - \theta^{Al}).$$

The two conditions above imply

$$(1 - \theta^{Ah}) + \Delta\theta_2^A\gamma \leq \omega_1^h - \omega_1^l \leq (1 - \theta^{Al}) + \mu^B\Delta\theta^B\gamma.$$

Such wages ω_1^h and ω_1^l only exist if $(1 - \theta^{Ah}) + \Delta\theta_2^A\gamma < (1 - \theta^{Al}) + \mu^B\Delta\theta^B\gamma$ which holds exactly when $\gamma < \Delta\theta^A/(\Delta\theta_2^A - \mu^B\Delta\theta^B)$.

Case ii) $\mu^B > \theta^{Bl}/\theta^{Bh}$: From Lemma 12, it then follows $U^h(h) = U^l(l) = 0$ and $U^h(h) = \Delta\theta_2\gamma$. Incentive constraint (20) therefore becomes:

$$\omega_1^h - (1 - \theta^{Ah}) \geq \omega_1^l + \Delta\theta_2^A\gamma.$$

Because in addition $U^l(h) = 0$, incentive constraint (21) becomes:

$$\omega_1^l \geq \omega_1^h - (1 - \theta^{Al}).$$

The two conditions above imply

$$(1 - \theta^{Ah}) + \Delta\theta_2^A\gamma \leq \omega_1^h - \omega_1^l \leq 1 - \theta^{Al},$$

which cannot be satisfied for any wages ω_1^h and ω_1^l, because $\theta_2^{Ah} > \theta^{Ah} > \theta^{Al}$ and $\gamma > 1$. ■

Appendix 2: Semi-Separating Strategies

We showed the optimality of vertical hierarchies by focusing on pooling and full separation contracts. It is well known however that, in general, optimal contract may also involve semi–separation. This is due to the limited commitment of the principal, which leads to a failure of the revelation principle. In this appendix we show that the superiority of vertical hierarchies is not due to our neglect of semi–separation. We, thereby, need to consider only semi separating strategies in the horizontal hierarchy, because we demonstrate that these outcomes lead to payoffs that are already lower than the principal's optimal contract in a full separating vertical hierarchy.

Bester and Strausz (2001) show that, despite imperfect commitment by the principal, direct mechanisms can implement any Pareto optimal outcome between the principal and the agent. These direct mechanisms induce the agent to report his type truthfully with a strictly positive probability, but may also require the agent to misreport with a strictly positive probability. In order to represent such mechanisms in our context, let $\alpha^a \in (0,1]$ denote the probability that agent A of type θ^a reports his type truthfully. A combination (α^h, α^l) represents agent A's reporting strategy when facing a direct mechanism. Because we can always relabel messages, we may also restrict attention to reporting strategies with $\alpha^h \geq 1 - \alpha^l$. Pooling contracts then coincide with the reporting strategy $\alpha^h = \alpha^l = 1/2$ and (full) separation contracts coincide with the reporting strategy $\alpha^h = \alpha^l = 1$. Because we focus in this appendix on reporting strategies that imply neither full pooling or full separation, we consider only reporting strategies from the set:

$$\mathcal{A} \equiv \{(\alpha^h, \alpha^l) \in (0,1) \times (0,1) \mid \alpha^h \geq 1 - \alpha^l\} \backslash \{(1/2, 1/2), (1,1)\}.$$

Moreover, denote the principal's posterior belief that agent A is efficient after he reports θ_A^a in $t = 1$ by μ_2^a. For a given reporting strategy α^h and α^l, the principal's beliefs μ_2^h and μ_2^l, in equilibrium, must satisfy Bayes' rule and:

$$\mu_2^h(\alpha^h, \alpha^l) \equiv \frac{\mu\alpha^h}{\mu\alpha^h + (1-\mu)\alpha^l} \text{ and } \mu_2^l(\alpha^h, \alpha^l) \equiv \frac{\mu(1-\alpha^h)}{\mu(1-\alpha^h) + (1-\mu)\alpha^l}. \tag{49}$$

For any reporting strategy $(\alpha^h, \alpha^l) \in \mathcal{A}$ expression (49) implies $\mu_2^l < \mu < \mu_2^h$.

In an equilibrium, the principal's offer in $t = 2$ is sequentially rational given agent A's reporting behavior. We can, therefore, view it as contingent on agent A's report θ^a and, by Lemma 1, also on agent B's type θ^b. The crucial question is whether the sequentially rational

contract itself is a pooling or a separation one. Therefore, let ρ^{ab} denote the probability that the principal offers a pooling contract in $t = 2$ given agent A's report θ^a and agent B's type θ^b. By Lemma 2, sequentially rational behavior of the principal implies:

$$\rho^{ab} \in \begin{cases} \{1\} & \text{if } \mu_2^a < \hat{\mu}(\theta^b), \\ [0,1] & \text{if } \mu_2^a = \hat{\mu}(\theta^b), \\ \{0\} & \text{if } \mu_2^a > \hat{\mu}(\theta^b). \end{cases} \tag{50}$$

From $\mu_2^l < \mu < \mu_2^h$, it then follows that $\rho^{lb} \geq \rho^{hb}$. This means that, in equilibrium, it is more likely that the principal offers a pooling contract if agent A reports himself as inefficient. It follows, because the subsequent decrease in the principal's belief that agent A is efficient shifts the trade–off between a pooling and a separation contract more in favor of pooling contracts.

On the equilibrium path, any sequentially rational contract in $t = 2$, leaves a rent to agent A only if the contract is a pooling one and, in this case, the rent is exactly $\Delta\theta\gamma$. Agent A of type θ^h therefore expects to receive this rent with probability $\mu\rho^{ah} + (1-\mu)\rho^{al}$ when he reports θ^a. Because, in equilibrium, agent A's reporting strategy (α^h, α^l) must be optimal, the reporting probability α^h satisfies:

$$\begin{aligned} \alpha^h \quad \in \quad & \arg\max_{\hat{\alpha}^h} \hat{\alpha}^h[\omega_1^h - (1-\theta^h) + (\mu\rho^{hh} + (1-\mu)\rho^{hl})\Delta\theta\gamma] \\ & + (1-\hat{\alpha}^h)[\omega_1^l + (\mu\rho^{lh} + (1-\mu)\rho^{ll})\Delta\theta\gamma]. \end{aligned} \tag{51}$$

Similarly, because agent A of type θ^l will never receive a rent in $t = 2$, the reporting strategy α^l, in equilibrium, satisfies:

$$\alpha^l \in \arg\max_{\hat{\alpha}^l} \hat{\alpha}^l\omega_1^l + (1-\hat{\alpha}^l)[\omega_1^h - (1-\theta^l)]. \tag{52}$$

With the help of α^h, α^l and ρ^{ab}, the principal's expected payoff in the beginning of $t = 1$ can be written as:

$$\begin{aligned} \Pi_{semi}^H \quad = \quad & \mu\{\alpha^h[1 - \omega_1^h + (\theta^h + \widetilde{\theta} - (\mu\rho^{hh} + (1-\mu)\rho^{hl})\Delta\theta)\gamma] \\ & +(1-\alpha^h)[-\omega_1^l + (\theta^h + \widetilde{\theta} - (\mu\rho^{lh} + (1-\mu)\rho^{ll})\Delta\theta)\gamma]\} \\ & +(1-\mu)\{\alpha^l[-\omega_1^l + (\mu\rho^{lh}(\theta^l + \theta^h) + (1-\mu)\rho^{ll}(\theta^l + \theta^l))\gamma] \\ & +(1-\alpha^l)[1 - \omega_1^h + (\mu\rho^{hh}(\theta^l + \theta^h) + (1-\mu)\rho^{hl}(\theta^l + \theta^l))\gamma]\}. \end{aligned} \tag{53}$$

Therefore, if semi–separation in the horizontal hierarchy is optimal, then there exists a combination $(\alpha^h, \alpha^l) \in \mathcal{A}$ that maximizes Π_{semi}^H subject to (49), (50), (51), and (52). In

order to solve this problem, we first show that, with semi–separation, optimal reporting strategies are necessarily such that they induce a belief of the principal that makes her exactly indifferent about offering a pooling or separation contract in $t = 2$. The intuition behind this result is that these beliefs bring the principal's myopic second period behavior in line with the principal's ex ante perspective of $t = 1$. Because, in equilibrium, the principal's beliefs depend on the agent's reporting strategy, his reporting behavior is a key instrument for controlling the principal's myopic behavior in $t = 2$.

Lemma 14 *If reporting behaviors in $\mathcal{A}$ are optimal, then there exist an optimal α^h and α^l such that i) $\alpha^h = 1$ and $\mu_2^h(1, \alpha^l) = \hat{\mu}(\theta^h)$, or ii) $\alpha^l = 1$ and $\mu_2^l(\alpha^h, 1) = \hat{\mu}(\theta^l)$, or iii) $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$.*

Proof. Suppose α^h and α^l are optimal and stochastic and $\mu_2^h(\alpha^h, \alpha^l) \neq \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) \neq \hat{\mu}(\theta^l)$. Because ρ^{ab} only changes at $\hat{\mu}(\theta^a)$, (53) is linear in α^h and α^l (for sufficiently small changes in α^h and α^l). However, the optimality of α^h and α^l implies that (53) can neither be increasing nor decreasing in α^h or α^l. Thus, (53) must be independent of α^h and α^l and we can increase or decrease α^h and α^l so that $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ or $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$. Because the original ρ^{ab} is consistent with these adapted α^h and α^l, it yields the same expected payoff to the principal and is hence also optimal. Suppose now there exist no deterministic reporting strategy at the optimum. Then, $\alpha^h \in (0, 1)$ and $\alpha^l \in (0, 1)$ so that $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ or $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$. If $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) \neq \hat{\mu}(\theta^l)$, then we can specify the implicit function $\widetilde{\alpha}^l(\alpha^h)$ by $\mu_2^h(\alpha^h, \widetilde{\alpha}^l(\alpha^h)) = \mu_2^h(\alpha^h, \alpha^l)$. Because $\widetilde{\alpha}^l$ is necessarily linear in α^h and (53) is linear in both α^h and α^l, replacing α_l with $\widetilde{\alpha}^l(\alpha^h)$ in (53) yields an expression for the principal's payoff $\widetilde{\Pi}^H_{semi}(\alpha^h)$ that is linear in α^h for a small change in α^h. Now, suppose $\mu_2^l(\alpha^h, \alpha^l) < \hat{\mu}(\theta^l)$, and consider the interval $[0, \bar{\alpha}^h]$, where $\bar{\alpha}^h$ is such that $\mu_2^l(\bar{\alpha}^h, \widetilde{\alpha}^l(\bar{\alpha}^h)) = \hat{\mu}(\theta^l)$. Note that if α^h and α^l maximize (53), then they must maximize $\widetilde{\Pi}^H_{semi}(\alpha^h)$ over $[0, \bar{\alpha}^h]$ at α^h, because ρ^{ab} remains unchanged for any combination $\{\alpha_h, \alpha^h, \widetilde{\alpha}^l(\alpha^h)\}$ with $\alpha^h \in [0, \bar{\alpha}^h]$. However, because $\widetilde{\Pi}^H_{semi}(\alpha^h)$ is linear in α^h, it must be maximized at either 0 or $\bar{\alpha}^h$. It follows that (53) must also be optimized for $\{\alpha^h, \alpha^l\} = \{0, \widetilde{\alpha}^l(0)\}$ or $\{\alpha^h, \alpha^l\} = \{\bar{\alpha}^h, \widetilde{\alpha}^l(\bar{\alpha}^h)\}$. The first case, $\{\alpha^h, \alpha^l\} = \{0, \widetilde{\alpha}^l(0)\}$, contradicts the assumption that there only exist optimal outcome in purely mixed strategy, implying that $\{\bar{\alpha}^h, \widetilde{\alpha}^l(\bar{\alpha}^h)\}$ must be optimal. Yet, as we needed to show, this implies $\mu_2^l = \hat{\mu}(\theta^l)$. If $\mu_2^l(\alpha^h, \alpha^l) > \hat{\mu}(\theta^l)$ similar arguments show that there must also exist optimal α^h and α^l with $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$. The case of $\mu_2^h(\alpha^h, \alpha^l) \neq \hat{\mu}(\theta^h)$ and

$\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$ can be treated similarly to show that there must then also exist optimal α^h and α^l with $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$. ■

With this characterization of reporting behavior, we may compute the principal's maximum payoff for the three different cases and compute her maximum payoff, Π_{semi}^H.

Proposition 8 *The payoff Π^V exceeds the payoff from any contract in the horizontal hierarchy that induces a semi–separating whenever $\mu < \mu^m$ and, in particular, for any $\mu < \hat{\mu}(\theta^h)$.*

Proof. We compute the payoffs for the three cases in Lemma 14. First, suppose $\alpha^h = 1$ and $\mu_2^h(1, \alpha^l) = \hat{\mu}(\theta^h)$ is optimal. Because $\mu_2^h > \mu$, this case exists only if $\hat{\mu}(\theta^h) > \mu$. Moreover, it holds $\mu_2^l = 0$ so that $\rho^{lh} = \rho^{ll} = 1$ and, due to $\mu_2^h(1, \alpha^l) = \hat{\mu}(\theta^h) > \hat{\mu}(\theta^l)$ it follows $\rho^{hl} = 0$ and the principal is indifferent between pooling and separation, and thus her expected payoff is independent of ρ^{hh}. Expression (53), therefore, simplifies to:

$$\begin{aligned}&\mu[1 - \omega_1^h + (\theta^h + \tilde{\theta})\gamma] \\ &+(1-\mu)\{\alpha^l[-\omega_1^l + (\mu(\theta^l + \theta^h) + (1-\mu)(\theta^l + \theta^l))\gamma] + (1-\alpha^l)[1 - \omega_1^h]\},\end{aligned}$$

and is to maximize under the incentive constraints,

$$\omega_1^h \geq (1 - \theta^h) + (1 - \mu\rho^{hh})\Delta\theta\gamma + \omega_1^l \quad \text{and } \omega_1^l = \omega_1^h - (1 - \theta^l),$$

and the participation constraints,

$$\omega_1^l \geq 0 \quad \text{and } \omega_1^h \geq 1 - \theta^h - \mu\rho^{hh}\Delta\theta\gamma.$$

Because ρ^{hh} does not influence the principal's expected payoff, but the constraints are unambiguously relaxed for larger ρ^{hh}, $\rho^{hh} = 1$ is optimal. As usual, the participation constraint for type θ^l together with the incentive constraint for type θ^h imply the participation constraint for type θ^h. It also follows that the two incentive constraints are consistent only if $(1-\mu)\gamma \leq 1$. In this case, $\omega_1^l = \omega_1^h - (1 - \theta^l)$ implies that the incentive constraint for type θ^h is satisfied, because it follows:

$$(1 - \theta^h) + (1-\mu)\Delta\theta\gamma + \omega_1^l = (1-\mu)\Delta\theta\gamma - \Delta\theta + \omega_1^h \leq \omega_1^h.$$

Hence, an optimal solution exists only if $(1-\mu)\gamma \leq 1$ and it exhibits:

$$\omega_1^h = 1 - \theta^l \text{ and } \omega_1^l = 0,$$

with the payoff:

$$\Pi^H_{semi} = \frac{2\mu\theta^h\theta^l + \left[(2\mu\theta^h - \mu^2\Delta\theta)\Delta\theta + 2\theta^l(\theta^h + \theta^l)\right]\gamma}{\theta^h + \theta^l},$$

which is smaller than Π^V for the entire implementable range $\mu < \hat{\mu}(\theta^h)$.

Suppose now $\alpha^l = 1$ and $\mu^l_2(\alpha^h, 1) = \hat{\mu}(\theta^l)$ is optimal. Because $\mu^l_2 < \mu$, this case exists only if $\hat{\mu}(\theta^l) < \mu$. Moreover, it holds $\mu^h_2 = 1$ so that $\rho^{hh} = \rho^{hl} = 0$ and, due to $\mu^l_2(\alpha^h, 1) = \hat{\mu}(\theta^l) < \hat{\mu}(\theta^h)$ it follows $\rho^{lh} = 1$ and the principal is indifferent between separation and pooling. Hence the principal's payoff in (53) is independent of ρ^{ll}, and simplifies to:

$$\begin{aligned}&\mu\{\alpha^h[1 - \omega^h_1 + (\theta^h + \widetilde{\theta})\gamma] + (1 - \alpha^h)[-\omega^l_1 + (\theta^h + \widetilde{\theta} - \mu\Delta\theta)\gamma]\}\\&+(1 - \mu)[-\omega^l_1 + \mu(\theta^l + \theta^h)\gamma],\end{aligned}$$

and is to maximize under the incentive constraints,

$$\omega^h_1 = (1 - \theta^h) + (\mu + (1 - \mu)\rho^{ll})\Delta\theta\gamma + \omega^l_1 \quad \text{and } \omega^l_1 \geq \omega^h_1 - (1 - \theta^l),$$

and the participation constraints,

$$\omega^l_1 \geq 0 \quad \text{and } \omega^h_1 \geq 1 - \theta^h.$$

Again, the participation constraint of θ^h is implied by the incentive constraint of θ^h and the participation constraint of θ^l. Moreover, because the principal's expected payoff is decreasing in ω^h_1, and ω^h_1 itself is decreasing in ρ^{ll}, a smaller ω^h_1 relaxes the incentive constraint for type θ^l, and it follows that $\rho^{ll} = 0$ must be optimal. It then follows that the two incentive constraints are consistent only if $\mu\gamma \leq 1$. Only in this case, an optimal solution exists and exhibits:

$$\omega^h_1 = (1 - \theta^h) + \mu\Delta\theta\gamma \text{ and } \omega^l_1 = 0,$$

with the payoff:

$$\Pi^H_{semi} = \frac{\mu{\theta^h}^2 - (2 - \mu)\theta^h\theta^l + \mu(2 - \mu)({\theta^h}^2 - {\theta^l}^2)\gamma}{\Delta\theta}.$$

This is smaller than Π^V for the range $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h)]$. For $\mu > \hat{\mu}(\theta^h)$, straightforward calculations show that $\Pi^H_{semi} > \Pi^V$ when

$$\mu > \frac{2\theta^h\theta^l}{(\theta^h + \theta^l)\Delta\theta\gamma}.$$

Finally, suppose $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h)$ and $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l)$ is optimal. Because $\mu_2^l < \mu < \mu_2^h$ this case exists only if $\hat{\mu}(\theta^l) < \mu < \hat{\mu}(\theta^h)$. Due to $\mu_2^h(\alpha^h, \alpha^l) = \hat{\mu}(\theta^h) > \hat{\mu}(\theta^l)$, it follows $\rho^{hl} = 0$ and, due to $\mu_2^l(\alpha^h, \alpha^l) = \hat{\mu}(\theta^l) < \hat{\mu}(\theta^h)$, it follows $\rho^{lh} = 1$. Moreover, the principal is indifferent to ρ^{hh} and ρ^{ll}, and hence her expected payoff must be independent of both ρ^{hh} and ρ^{ll}. The expression in (53) simplifies to:

$$\begin{aligned}&\mu\{\alpha^h[1-\omega_1^h+(\theta^h+\widetilde{\theta})\gamma]+(1-\alpha^h)[-\omega_1^l+(\theta^h+\widetilde{\theta}-\mu\Delta\theta)\gamma]\}\\&+(1-\mu)\{\alpha^l[-\omega_1^l+\mu(\theta^l+\theta^h)\gamma]+(1-\alpha^l)[1-\omega_1^h]\},\end{aligned} \tag{54}$$

and is to maximize under the incentive constraints,

$$\omega_1^h = (1-\theta^h) + (\mu(1-\rho^{hh}) + (1-\mu)\rho^{ll})\Delta\theta\gamma + \omega_1^l \quad \text{and } \omega_1^l = \omega_1^h - (1-\theta^l),$$

and the participation constraints,

$$\omega_1^l \geq 0 \quad \text{and } \omega_1^h \geq 1-\theta^h + \mu\rho^{hh}\Delta\theta\gamma.$$

The incentive constraints are consistent only if $\mu(1-\rho^{hh}) + (1-\mu)\rho^{ll} = 1/\gamma$. This implies $\omega_1^h = 1-\theta^l+\omega_1^l$. Substituting out the wage ω_1^h in (54) demonstrates that the expression is decreasing in ω_1^l. The optimal wage is $\omega_1^l = 0$, which satisfies the participation constraint of type θ^h for $\rho^{hh} = 0$. It yields the principal the payoff:

$$\Pi_{semi}^H = \frac{2\theta^h\theta^l(\mu(\theta^h+\theta^l)-2\theta^l)+\mu(2-\mu)(\theta^h+\theta^l)\Delta\theta^2\gamma}{\Delta\theta^2},$$

which is smaller than Π^V for the entire implementable range $\mu \in (\hat{\mu}(\theta^l), \hat{\mu}(\theta^h))$. ■

The proposition above states that, for $\mu \in (\hat{\mu}(0), \hat{\mu}(\theta^h))$, the vertical hierarchy is also optimal when considering semi–separating strategies. For $\mu \in (\hat{\mu}(\theta^h), 1)$, the region of vertical hierarchy in Figure 1 survives unless μ is very high. The intuition is as follows. With the semi–separating strategy, the probability of zero output in both periods is strictly positive. However, there is a chance that separation takes places in $t = 1$ with no rent provision in $t = 2$. Thus, if it is highly likely that the agent is efficient, then the principal prefers to take a risk in $t = 1$ by inducing the agent's randomization. By doing so the principal's cost to separate the types of agent A in $t = 1$ becomes lower. If, however, the agent is not highly likely to be efficient, then the principal is better off by avoiding the semi–separating strategy.

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