# Foreign Firms and the Diffusion of Knowledge\*

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#### Abstract

This paper constructs a model to examine the impact of foreign firms on a developing country's own accumulation of entrepreneurial knowledge. In the model, entrepreneurial skills are built up on the basis of productive ideas that diffuse internally (at the inside of firms) and externally, via spillovers. Openness to foreign firms enhances the aggregate exposure to ideas but also reduces the returns to investing in entrepreneurial skills. When externalities are present, openness can be welfare reducing. However, regardless of the relative importance of externalities, simple quantitative exercises suggest that the gains of openness are positive and can be large.

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### 1 Introduction

Entrepreneurial knowledge, the know-how to combine technology and market opportunities to set up and manage firms, can be the limiting factor in a country's aggregate productivity. Countries with a short supply of these skills can import them from more developed countries and recent work by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and Eeckhout and Jovanovic (2010) suggests that developing countries may accrue significant output and consumption gains by hosting foreign firms. But, does the presence of foreign firms enhance or impair the domestic accumulation of know-how? Would hosting foreign firms lead a developing country to catch up with developed countries or to lag further behind? Once the effect on domestic skills is accounted for, what is the impact of this form of openness on the overall welfare of a country? This paper uses a simple general equilibrium growth model to answer these questions.

The model is as follows: Entrepreneurs lead firms, production teams of workers and midmanagers. As in Lucas (1978), the knowledge of the entrepreneur determines the productivity of the team. The model is an OLG economy in which some of the young build up knowledge to set, manage and be the residual claimant of a firm when old. Knowledge, the engine of growth in the economy, has a dual nature. On one hand, it is a rival factor, the skills of an individual with a limited span-of-control on production activities. On the other hand, knowledge may also be a non-rival factor, productive ideas that once implemented could be used by anybody building up skills in the country. In a "closed" country only national entrepreneurs can set up firms; in an "open" country foreign firms are free to enter.<sup>3</sup> The entry of foreign firms not only impacts the set entrepreneurial skills implemented in a country but also the set of productive ideas available to its young generation.

The ideas upon which a young entrepreneur builds up his know-how come from two sources: the specific know-how running the firm in which he is a worker and the productive ideas implemented by the entire set of firms operating in the country. In this way, the model encompasses as special cases two common —but conflicting— views of the accumulation and diffusion of knowledge. In one extreme, the young individual's own firm is the only source of ideas, as in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995) and similar to Boldrin and Levine (2009). In the other extreme, the productive ideas implemented by each firm are uniformly exposed to all the young in the country. Variants of such assumption have a dominant presence in the literature on growth (e.g. Romer 1986, Klenow 1998 and Jones 2006), the impact of openness to trade on growth (e.g. Stokey 1991) and the impact of openness to multinational firms (e.g. Findlay 1978). In this paper both 'internal' and 'external' sources of ideas are present and an 'internalization' parameter determines their relative importance. Inasmuch as internalization is only partial, another 'external diffusion' parameter determines how easily the more advanced ideas can contribute the formation of skills in a country.

The equilibrium in the labor market is characterized by relationships between young and old

<sup>&</sup>lt;sup>1</sup>An long line in the literature links the productivity of firms to the quality of their management, e.g. Kaldor (1934), Lucas (1978), Rosen (1982), Prescott and Visscher (1980), Garicano (2000) and Bloom and Van Reenen (2007).

<sup>&</sup>lt;sup>2</sup>The gains are even larger if these skills are non-rival factors, i.e. can be used simultaneously in many locations. See Ramondo (2008) McGrattan and Prescott (2008).

<sup>&</sup>lt;sup>3</sup>The emphasis on the cross-border reallocation of management conforms with the observation that multinational firms heavily rely on home expatriates –and home trained individuals– to manage their operations, specially in developing countries (see Chapters 5 and 6 of UNCTAD 1994). It also conforms with the emphasis of the literature on firm specific intangible assets (e.g. Barba-Navarretti 2004 and Markusen 2004).

entrepreneurs that internalize the learning opportunities of firms with different knowledge levels. On the basis of the available learning opportunities, each young entrepreneur builds up his own skills foreseeing the set of skills with which he will be competing against. Even for a closed economy, the endogenous formation of skills leads to non-trivial dynamics, but simple conditions are provided upon which the accumulation of entrepreneurial knowledge is an engine of sustained growth and the country exhibits a balanced-growth-path (BGP).

Entry of foreign entrepreneurs impacts the accumulation of skills of the host country in three ways. First, foreign firms enhance the exposure to ideas of the domestic young directly working for them. Second, foreign firms may have positive externalities (spillovers) on the set of ideas circulating in the country, which benefits all the local young, including those working for domestic firms. While these two are positive effects, a third one is detrimental: foreign entrepreneurs bid up the cost of labor in the country for all future periods, reducing the returns and the incentives of domestic entrepreneurs to invest in know-how. When externalities impact the formation of skills, openness to foreign firms can reduce the formation of domestic knowledge and be welfare reducing.

Open economies may exhibit an interesting vintage structure for the population of domestic firms. With less that perfect internalization, the domestic entrepreneurs who build up their skills working for foreign firms do not fully catch up with their foreign counterparts. The young entrepreneurs working for them will lag further behind. Endogenously, each vintage will fall below, in relative terms, the previous vintage. This equilibrium structure is similar to that of Chari and Hopenhayn (1991), but includes two important additional aspects: First, the productivity levels of each vintage is endogenous. Second, the set of productive ideas circulating in the country is also endogenous, determined by domestic investments and foreign entry.

The model implies that gains from openness are regressive, i.e. higher for the more backward countries.<sup>4</sup> Openness enables backward countries to build up skills on the basis of the more advanced knowledge of developed countries and this enhanced exposure to ideas more than compensates the negative effect of a higher price for (unskilled) labor. Interestingly, openness can lead to *leapfrogging* among developing countries. If the complementarity between domestic and foreign sources of ideas is not too strong, i.e. if foreign ideas can be understood despite the low local knowledge (in a sense explained in the paper), then, upon openness, young entrepreneurs from more backward countries end up better exposed to productive ideas overall. In these circumstances, when two developing countries open, the initially more backward country will surpass and consistently remain ahead of the other, although both countries eventually converge to the same position in the BGP.

The gains from openness are even stronger if occupation choices are introduced. Allowing individuals to choose between managerial and labor occupations can enhance the static gains of openness as shown by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully by Eeckhout and Jovanovic (2009). With endogenous skill formation, occupation choices can enhance the gains of openness further. Not only they can change the structure of the BGPs towards more productive ones, but they can also redirect an open economy away from a laggard (interior) BGP and towards fully catching up with developed countries. Occupation choices can also accelerate the convergence. Introducing occupation choices leads to a different, starker form of leapfrogging: After openness, a more backward country may end up fully catching up with developed countries while the initially more advanced remains forever behind (in the interior BGP). This form of leapfrogging is independent and very different from the previous one, but both enhance the gains of openness more for the more backward countries.

<sup>&</sup>lt;sup>4</sup>This is the opposite from the of gains of openness to trade in Stokey (1991) and Young (1991).

In order to explore further the gains of openness for a developing country, I extend the basic model to restrict the formation and diffusion of entrepreneurial skills to a small fraction of young individuals who directly interact with the decision-makers at the top of the firms. The extensions also allow me to use values for the span-of-control parameter traditionally used for quantitative exercises. Then, I simulate the response and welfare consequences for different initial conditions and various specifications of the internalization and external diffusion parameters. These simple exercises generate surprisingly conclusive messages. First, as long as span-of-control parameter values are in the range used in the literature, openness lead developing countries very close to fully catching up with developed countries. This holds regardless of the internalization and external diffusion parameters and does not depend on occupation choices. I find that openness can only push back countries that are already very closed to the developed leaders, and even for them, the losses are negligible. Thus, to the question of whether openness lead developing countries to catch up, the answer is, absent other barriers or frictions, yes.

Second, the aggregate welfare gains from openness can be very high, huge indeed for countries that lag far behind. Even accounting for the costs of building up the skills, welfare gains are comparable to the (cross-BGPs) output gains since domestic entrepreneurial knowledge accrue in the form of higher national income and consumption and also in lower costs of building up future skills. The theoretical losses of openness pointed above hold only for countries close to the frontier and are negligible. Once other potential benefits of openness for developed countries are considered, the results of this paper strongly point in favor of openness for all countries. Other interesting aspects are: (i) the gains of openness with endogenous formation of knowledge are generally larger than the (static) gains holding skills fixed; this holds even under very severe limitations for the impact of foreign knowledge in the country; and (ii) the global behavior of the gains of openness is very similar even across parameter specifications that lead to very different mechanisms and dynamics in the diffusion of foreign knowledge.

The welfare gains from openness can be larger when the diffusion of ideas is fully internalized (and hence foreigners are compensated for it) than when it is via externalities. This is even more interesting because the observable implications of the model are consistent with empirical evidence only when at least some of the diffusion of knowledge is internalized. First, with internalized diffusion, openness to foreign knowledge pushes pre-existing domestic to reduce their productivity, as documented for developing countries by Aitken and Harrison (1999), Xu (2000), and Alfaro et. al (2006). Second, only when diffusion is at least partially internalized the model is consistent with the emergence of new domestic sector as described by Rhee and Belot (1990) in Bangladesh, Colombia and Indonesia.<sup>5</sup>

Recent work by Beaudry and Francois (2010) also study the impact of openness on the accumulation of managerial skills. They assume that individuals can be of either two types, skilled or unskilled and that the probability of an unskilled person to become skilled depends on the number of skilled workers in his firm. Interestingly, in this case, the gains of a skilled worker to migrate to a developing country may be zero or negative because his teaching contribution may be higher in the developed country than in the developing one. On the contrary, Dasgupta (2009), assumes that transfers of knowledge are exogenous but stochastic, generating sustained heterogeneity across workers and managers. The distribution of skills is continuous and the equilibrium exhibits positive-assortive matching. Interestingly, Dasgupta finds that the response to openness can be Pareto improving, as low skilled domestic firms provide early training for the future work-

<sup>&</sup>lt;sup>5</sup>For support of internal diffusion at the industry level, see Keppler (2001, 2002, 2006) for the car industry and Agarwal et al (2004), Filson and Franco (2006) and Franco (2005) for the rigid disk drive industry.

ers of more advanced foreign firms. In both papers, however, the analysis is restricted to exogenous and fully internalized transfers of knowledge. In this paper, fully internalized transfers are just a special case since externalities are also allowed to affect the formation of skills. Moreover, the endogeneity of the investment in skills across firms with different learning opportunities is a key aspect of the response to openness. Since in this paper knowledge is the engine of growth for all countries, how domestic knowledge reacts to the inflow of foreign knowledge determines whether diffusion is also the engine of multinational activity.

To focus on the accumulation of entrepreneurial knowledge, I have abstracted from many aspects studied in the literature of multinational activity such as the endogenous choice of organization (see the recent survey by Antras and Rossi-Hansberg 2009 and references therein), and the choice of technologies that multinational firms send to their subsidiaries (e.g. Helpman 1984 and Keller and Yeaple 2010). The analysis also abstracts from international worker mobility (e.g., Rauch 1991; Klein and Ventura 2006), and from interactions between technology diffusion, multinational activity and international trade in goods (e.g. Grossman and Helpman 1991, Eaton and Kortum 2006, Rodriguez-Clare 2007, and Alvarez, Buera, and Lucas 2010). The paper also omits other forms of knowledge or human capital (e.g. Krishna and Chesnokova 2009) and their interaction with technology adoption (e.g. Stokey 2010), and does not consider specificity or appropriateness of technologies (e.g. Basu and Weil 1998). I have also abstracted from cross-country spillovers (e.g. Damsgaard and Krusell 2008 and Klenow and Rodriguez-Clare 2005) and flows of physical capital (Gourinchas and Jeanne 2003) Finally, the paper assumes that there are no frictions or tax distortions at the interior of countries in the allocation of workers across managers (e.g. Buera and Shin, 2010, Cagetti and De Nardi, 2006, Guner et al. 2008 among others).

The rest of the paper proceeds as follows. In Section 2, I set up the basic model, where for simplicity I assume that all the young have the potential to be entrepreneurs when old. In Section 3, I characterize and compare the competitive equilibria of closed and open economies and derive some basic messages about the gains from openness. Section 4 introduces occupation choices and their impact on the gains from openness. Section 5 lays out the extended model in which production involves entrepreneurial knowledge and mid-management and labor services and only a fraction of the young have entrepreneurial potential. This model is then used for the quantitative exercises. Appendixes at the end develop analytical details of both the basic and the extended model.

### 2 The Basic Model

Consider a discrete time, infinite horizon OLG economy with a single consumption good and individuals that live for two periods. The utility of an individual born at time t that consumes  $c_t^t$  and  $c_{t+1}^t$  in periods t and t+1 is

$$U^t = c_t^t + \beta c_{t+1}^t,$$

where  $0 < \beta < 1$ .

The size (measure) of all generations is equal to one. All individuals have an endowment of one unit of time every period they are alive. When young, individuals supply their time endowment as labor; when old, they could use their time to become "entrepreneurs", i.e. set up and control a firm. Yet, entrepreneurship is just an option; old individuals can remain workers. The value of their 'career' options are foreseen by the young and they decide whether and how much to invest in acquiring entrepreneurial know-how.

As in Lucas (1978), the consumption good is produced by 'firms', teams of one entrepreneur (or manager) and a group of workers. The entrepreneur is the residual claimant of the single firm he sets up and manages.<sup>6</sup> The (person-specific) skills or knowledge z of the entrepreneur determines the productivity of the firm under his control. With z units of entrepreneurial skills and n units of labor, a firm produces

$$y = zn^{\alpha}$$
,

units of the consumption good. The span-of-control parameter  $\alpha \in (0,1)$  is the degree of decreasing returns to the amount of labor n.<sup>7</sup>

The core of the analysis is in the accumulation and diffusion of entrepreneurial know-how. Therefore, unlike much of the existing span-of-control models (e.g. Lucas 1978, Cagetti and De Nardi, 2006, Burstein and Monge-Naranjo 2009 or Buera and Shin 2010,), entrepreneurial skills cannot be taken as an exogenously distributed endowment of talent. Instead, they must be endogenously determined as the outcome of optimal investments, the maximization of a young person's foreseen returns of setting up and controlling firm minus the cost of building up the required skills. Both, the costs and the returns of entrepreneurial skills are determined in equilibrium, as I now proceed to explain. The costs are determined by the set of productive ideas a person is exposed to when young; the returns, by the skills of the other competing entrepreneurs when old.

The exposure to ideas of a young person, denoted  $z^E \geq 0$ , subsumes the contributions of two sources: (i) the productive know-how z of the entrepreneur that controls the firm where the young individual is a worker, and (ii) an average  $Z^O$  of the know-how of all the entrepreneurs actively operating inside the country at that time. Therefore,  $z^E = F(z, Z^O)$ , where  $F: \mathbb{R}^2_+ \to \mathbb{R}_+$  is a positive and linearly homogeneous function that is increasing and twice differentiable in both arguments. For my purposes, the Cobb-Douglas case suffices:

$$z^{E} = (z)^{\gamma} \left( Z^{O} \right)^{1-\gamma}, \tag{1}$$

where  $0 \le \gamma \le 1$  will be called the "internalization" parameter because it determines how much a person learns inside his own job relative to how much he learns from outside.

The average  $Z^O$  summarizes the set of productive ideas outside each individual's firm. It is a national "public good", i.e. a non-rival factor to which everyone in the country has free access to.<sup>8</sup> It is determined as follows: Let  $\mu_z$  be the (endogenous) probability measure that indicates the allocation of the country's total labor across the firms with different know-how levels. That is, for any Borel set  $B \subset \mathbb{R}_+$ ,  $\mu_z(B)$  indicates the share of the labor in control of entrepreneurs with know-how levels in B. Then,  $Z^O$  is a generalized (or Hölder) weighted mean of all the active firms:

$$Z^{O} = \left[ \int_{\mathbb{R}_{+}} (z)^{\rho} \,\mu_{z} \,(dz) \right]^{\frac{1}{\rho}}, \tag{2}$$

<sup>&</sup>lt;sup>6</sup>This formulation of equilibrium is equivalent to one in which firms with constant returns to scale (and zero-profits in equilibrium) are the ones hiring "managerial" services from the entrepreneurs. For a model that distinguishes between the economic functions of entrepreneurs and managers, see Holmes and Schmitz (1994).

<sup>&</sup>lt;sup>7</sup>As Lucas (1978), I call these teams "firms" even if they can equally be seen as parts of a conglomerate of teams within the boundaries of the same firm. However, see Garicano (2000), Oi (1983) and Rosen (1982) for related issues

<sup>&</sup>lt;sup>8</sup>Notice the dual nature of entrepreneurial knowledge. On one hand, as in Boldrin and Levine 2009, knowledge are skills, and as such, a rival factor that is tied to the time of the holder; it cannot be used simultaneously in multiple tasks. On the other hand, as in Romer 1986, knowledge are ideas; as long as  $\gamma > 1$  they are a non-rival, partially non-excludable factors that could be used by any young forming entrepreneur in the country without crowding out the use by others.

where the parameter  $\rho$  can assume any value in the extended real numbers. This formulation encompasses many familiar ways of averaging the the know-how levels of active firms in a country. If  $\rho \to -\infty$ ,  $Z^O$  is the minimum value (Leontieff function); if  $\rho \to \infty$ , it is the maximum value. The arithmetic, geometric and harmonic means correspond to, respectively,  $\rho = 1, 0, -1$ .

Given the exposure to productive ideas  $z^E$ , the cost (in terms of current consumption) for a young individual to acquire a next-period level of skills  $z' \geq 0$  is  $z^E \phi\left(z'/z^E\right)$ , where  $\phi: \mathbb{R}_+ \to \mathbb{R}_+$  is a non-negative, continuously differentiable and strictly convex function with  $\lim_{x\to 0} \phi\left(x\right) = \phi'\left(x\right) = 0$  and  $\lim_{x\to\infty} \phi\left(x\right) = \phi'\left(x\right) = \infty$ . Then, the total and marginal costs of investing are strictly increasing and strictly convex in z' and strictly decreasing in  $z^E$ . It is convenient to focus on the special case

$$\phi\left(\frac{z'}{z^E}\right) = \frac{v_0}{1+v} \left(\frac{z'}{z^E}\right)^{1+v},\tag{3}$$

where  $v_0$ , v > 0. The marginal cost of z' is  $\phi'(z'/z^E) = v_0(z'/z^E)^v$ , which depends only one the ratio  $z'/z^E$ , i.e. how far an individual accumulates skills relative to his exposure to ideas  $z^E$ . I shall keep using  $\phi(\cdot)$  and  $\phi'(\cdot)$  as shorthands in some of the formulas below.

The parameters  $\rho$ , v and  $\gamma$  are key for the diffusion of know-how. The *curvature* parameter v determines the impact of  $z^E$  on the costs of acquiring z'; it determines how quickly know-how will grow over time and will also play an important role for equilibrium cross-firms differences. The diffusion parameter  $\rho$  determines how easily superior ideas diffuse inside a country. The higher the value of  $\rho$ , the higher the impact of superior ideas on the common pool  $Z^O$ . In the extreme, if  $\rho = +\infty$ , only the very best of all the ideas are considered in  $Z^O$ . In the opposite extreme, a value  $\rho = -\infty$ , implies that only the worst ideas are understood and can be used to build up skills.

Most importantly, by allowing any value  $0 \le \gamma \le 1$ , the model encompasses two common –but conflicting– views of the accumulation and diffusion of knowledge. On one hand, if  $\gamma = 0$ , then a common value  $z^E = Z^O$  holds for everyone and externalities are the only engine of accumulation and diffusion. Such assumption has a dominant presence in the literature on growth (e.g. Romer 1986 and Lucas 1988), the impact of openness to trade on growth (e.g. Stokey 1991) and the impact of openness to multinational firms (e.g. Findlay 1978). On the other hand, if  $\gamma = 1$ , then the exposure to ideas –and hence, the ability to accumulate skills– are uniquely determined by one's own firm. This gives rise to a richer relationship between young and old entrepreneurs, one that fully internalize the costs and benefits of accumulating skills. Such is the view in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995), and others. By allowing any  $0 \le \gamma \le 1$ , the model here combines the impact of externalities with labor markets that compensate for differences in the learning opportunities across firms with different knowledge levels.

I consider two types of economies. In a closed economy, only domestic entrepreneurs can set up firms and hire local labor; the demand for labor of the domestic old must equal the supply of labor from the domestic young. In an open economy, foreign entrepreneurs are allowed free entry; a free entry condition determines how much of the local labor is hired by foreign firms operating in the country.

## 3 The Formation and Diffusion of Knowledge and Skills

In this section I construct the objects needed to define a competitive equilibrium and then characterize and compare the accumulation of knowledge in closed and open economies.

#### 3.1 Equilibrium preliminaries

I consider perfect foresight competitive equilibria. The discount factor  $\beta$  pins down the interest rate with which all future payoffs are discounted. The key component of the price system is a sequence of wage functions  $\{w_t : \mathbb{R}_+ \to \mathbb{R}_+\}_{t=0}^{\infty}$ . The wage  $w_t(z)$  indicates the price that an entrepreneur with skills z must pay for a unit of labor at time t. The dependence of wages on the skills of the entrepreneur is driven by the learning opportunities offered by the firm under his control as explained below.

The economic decisions of an old person are whether to remain a worker or become an entrepreneur and, if an entrepreneur, how much labor to hire. As in Lucas (1978) this occupation choice is made on the basis of the individual's own skills z and the market wage; the difference is that the wage w(z) may depend on his own z. Should be become an active entrepreneur, the net rents for such old person would be

$$\pi [z, w (z)] \equiv \max_{\{n\}} \{zn^{\alpha} - w (z) n\}$$

$$= \theta z^{\frac{1}{1-\alpha}} [w (z)]^{\frac{-\alpha}{1-\alpha}}, \qquad (4)$$

where  $\theta \equiv (1 - \alpha) \alpha^{\frac{\alpha}{1 - \alpha}} > 0$ . Given w(z),  $\pi[\cdot, w(z)]$  is strictly increasing and convex; given z,  $\pi[w(z), \cdot]$  is strictly decreasing in w(z). Likewise, the optimal demand for labor would also be increasing in z and decreasing in w(z):

$$n^* [z, w(z)] = \left[ \frac{\alpha z}{w(z)} \right]^{\frac{1}{1-\alpha}}.$$
 (5)

An old individual become an active entrepreneur if and only if the rents  $\pi\left[z,w\left(z\right)\right]$  dominate the entire schedule of wages available to workers, which is determined by the function  $w\left(\cdot\right)$  and the entire set of active firms operating in the country.

The economic decisions of a young person are first, selecting the firm for which to work and second, conditional on that decision, whether and how much to invest in entrepreneurial skills. With respect to the latter, given the exposure to ideas  $z^E$  and the next period's cost of labor  $w_{t+1}(\cdot)$ , the optimal investment in entrepreneurial skills z' solves

$$V\left[z^{E}, w_{t+1}\left(\cdot\right)\right] \equiv \max_{z'} \left\{\beta \pi \left[z', w_{t+1}\left(z'\right)\right] - z^{E} \phi \left(z'/z^{E}\right)\right\}. \tag{6}$$

The key determinant of the optimal z' are  $z^E$  and  $w_{t+1}(\cdot)$ . Under the conditions laid out in Proposition 1 below, optimal investments in skills are determined by the condition

$$\beta \left[ \pi_1 (z', w_{t+1}(z')) + \pi_2 (z', w_{t+1}(z')) \frac{\partial w_{t+1}(z')}{\partial z'} \right] = \phi' \left( \frac{z'}{z^E} \right), \tag{7}$$

where  $\pi_1(\cdot)$  and  $\pi_2(\cdot)$  stand for, respectively, the first derivative of  $\pi$  with respect to the skill z of the manager and the wage  $w_{t+1}(z)$  he will have to pay for labor.

Most obviously, a better exposure of ideas (i.e. a higher  $z^E$ ) reduces directly the total and marginal costs of investment and leads to a higher z'. Indeed, as discussed below, if  $z^E$  is too low, the corner solution of of z' = 0 might be optimal. Young workers of very backward firms will also be workers when old.

With respect to the future wage function  $w_{t+1}(\cdot)$  notice that it impacts the optimal choice z' in two ways. First, there is a direct impact  $(\pi_1 > 0)$  since higher values of  $w_{t+1}(\cdot)$  reduce profits

for all levels of z'. Second, there is an indirect impact  $(\pi_2 < 0)$  since a negatively-sloped  $w_{t+1}(\cdot)$  reduces the cost of labor for the more skilled entrepreneurs, increasing their profits.

Differences in learning opportunities are fully perceived by all young individuals when choosing which firms to work for. As in Chari and Hopenhayn (1991), for simplicity, I am assuming that all young individuals are identical. Then, in equilibrium all workers must be indifferent to work for the different active firms as wages must compensate for differences in learning opportunities. Two active firms with arbitrary know-how levels  $z_0 < z_1$  pay wages that satisfy:

$$w_t(z_0) - w_t(z_1) = V\left[z_1^E, w_{t+1}(\cdot)\right] - V\left[z_0^E, w_{t+1}(\cdot)\right],$$
 (8)

where  $z_0^E = (z_0)^{\gamma} (Z^O)^{1-\gamma} < z_1^E = (z_1)^{\gamma} (Z^O)^{1-\gamma}$ . Less skilled managers must pay higher wages as the right-hand-side of this equation is positive. It is important to keep in mind that the proper interpretation of (8) is as differences in the cost of effective units of labor across firms, which may not translate easily to differences in earnings with heterogeneity across these units across workers.

Occupation choices, i.e. the option of old individuals to forsake entrepreneurship and remain workers can completely reshape the equilibrium of both open and closed economies, either by changing the set of steady states (balance-growth-paths) or the transition dynamics. However, for clarity of exposition, it is convenient to defer their analysis to a separate section (Section 4). Therefore, in the reminder of this section I proceed under the assumption that all the individuals are entrepreneurs when old and all of them invest in skills when young following (7).

#### 3.2 Skill formation in closed economies

Consider a closed economy. Since, only the domestic old can supply the entrepreneurial skills and operate firms in the country, they are the sole source of entrepreneurial ideas for the domestic young generation. Let the probability measure  $\phi_z^t$  describe the distribution of skills of the old generation in period t. Assume that  $\phi_z^t$  has a strictly positive support  $[z_L^t, z_H^t] \subset \mathbb{R}_+$  that is bounded from above. Given wages  $w_t(z)$ , the amount of labor  $n_t^*(z)$  hired by an entrepreneur with skill level z is given by (5), and the distribution of labor employed across skill levels is given by  $\mu_z^t(B) = \left(\int_B n_t^*(z) \phi_z^t(dz)\right) / \int_{z_L^t}^{z_H^t} n_t^*(z) \phi_z^t(dz)$  for any Borel set  $B \subset [z_L^t, z_H^t]$ . With  $\mu_z^t$  thus determined, the exposure to external ideas,  $Z_t^O$ , is determined as in expression (2), and the total exposure  $z_t^E$  of ideas of a young working in firm with know-how z is  $(z)^{\gamma} \left(Z_t^O\right)^{1-\gamma}$ .

To examine the accumulation of skills, derive expression (4),  $\pi_1 = \left[\alpha z/w\left(z\right)\right]^{\frac{\alpha}{1-\alpha}}$ , and  $\pi_2 = -\left[\alpha z/w\left(z\right)\right]^{\frac{1}{1-\alpha}}$  and plug them in (7). Then, from equilibrium condition (8), it follows that  $\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}\left(z''\right)\right] \frac{\partial \left(z^E\right)'}{\partial z'}$ . The envelope condition of (6) implies that

$$V_{1}\left[\left(z^{E}\right)', w_{t+2}\left(z''\right)\right] = -\phi\left(\frac{z''}{\left(z^{E}\right)'}\right) + \left(\frac{z''}{\left(z^{E}\right)'}\right)\phi'\left(\frac{z''}{\left(z^{E}\right)'}\right),$$

$$= -\frac{vv_{0}}{1+v}\left(\frac{z''}{\left(z^{E}\right)'}\right)^{1+v},$$

<sup>&</sup>lt;sup>9</sup>For instance, an economy with heterogeneous entrepreneurs and heterogeneous workers and small fixed costs of hiring each worker. More productive firms would want to hire more units of effective labor, and to minimize on the fixed costs, in equilibrium they would hire the workers endowed with the most effective units. Such positive assortive matching could lead to higher earnings for workers in the more productive firms.

where  $(z^E)'$  and (z'') are, respectively, the the next period's exposure of ideas and the investments for the workers of the next period entrepreneurs. Notice also that the second line uses the functional form assumed for  $\phi(\cdot)$ .

With all of this, after simplifying, equation (7) becomes

$$\beta \left( \frac{\alpha z'}{w'(z')} \right)^{\frac{\alpha}{1-\alpha}} \left[ 1 + \frac{\alpha v v_0 \gamma}{(1+v)} \frac{\left(z'\right)^{\gamma} \left(Z^{O'}\right)^{1-\gamma}}{w'(z')} \left( \frac{z''}{\left(z^{E}\right)'} \right)^{1+v} \right] = v_0 \left( \frac{z'}{z^{E}} \right)^{v}. \tag{9}$$

Based on this expression, Appendix A contains the proof of the following result:

**Lemma 1** The equilibrium wage function  $w_t(z')$ , if it exists, is non-increasing. For positive values  $Z^O$ ,  $(Z^O)'$  assume that a function z'(z) indicates the optimal investments in skills z' given current z. Then, if  $v > \alpha/(1-\alpha)$ , <sup>10</sup> the function  $z'(\cdot)$  is strictly increasing. Additionally, if  $\gamma > 1 - \frac{\alpha}{(1-\alpha)v}$ , then  $z'(z_1)/z'(z_0) > z_1/z_0$  for any  $z_1 > z_0$ .

Albeit limited, this simple result has important implications for the limiting behavior of the skill distribution. Most interestingly, if  $\gamma > 1 - \alpha/\left[(1 - \alpha)v\right]$ , i.e. one's own manager is a leading source of ideas, then pre-existing differences in the exposure to ideas will lead to widening gaps in skill formation. Yet, precisely because of these widening gaps, as explained below, the economy will eventually converge to a homogeneous pool of entrepreneurs.

In any event, regardless of the value of  $\gamma$ , if at any point in time the old generation of entrepreneurs is homogenous, so will be all the subsequent generations. To see this, assume that all old individuals possess the same level of know-how Z > 0. Then, in equilibrium, all the young individuals must receive the same wage w, are exposed to the same level of ideas  $z^E = Z$ , and will invest the same amount in skills Z'. In the next period, their own workers will be exposed to the same  $(z^E)' = Z'$ , and will receive the same wage w'. And so on. In every period, all firms will hire the same units of labor,  $n^* = 1$ , and market-clearing wages and earnings for entrepreneurs are  $w_t = \alpha Z_t$  and  $\pi_t = (1 - \alpha) Z_t$ . Imposing these conditions, equation (9) implies that the growth in skills  $G_t \equiv Z_{t+1}/Z_t$  must satisfy the difference equation

$$\beta \left[ 1 + \frac{\gamma v v_0}{(1+v)} \left( G_{t+1} \right)^{1+v} \right] = v_0 \left( G_t \right)^v. \tag{10}$$

A balance growth path (BGP) is an equilibrium in which entrepreneurial knowledge grows at a constant rate, i.e.  $G_t = G > 0$  all t. For this to hold, G must be a root of the equation (10) when  $G_{t+1} = G_t = G$ . Observe that for a BGP to exist the curvature parameter v must be high enough. Otherwise, the left-hand-side could always lay above the right-hand-side, and skill accumulation would degenerate to  $+\infty$ . As detailed in Appendix A, there are at most two roots, and in such a case only the lower root is relevant. Moreover, self-fulfilling (extrinsic) fluctuations are ruled out. These statements are summarized in the following proposition which is proved in the appendix.

**Proposition 1** (Closed economy BGP) For a closed economy: (a) An equilibrium BGP with homogeneous skills exists if either (i)  $\gamma > 0$ ,  $v > 1/(1-\alpha)$  and  $\beta \le (v_0/[\gamma^v(1+v)])^{\frac{1}{1+v}}$  or (ii)  $\gamma = 0$  and  $\gamma > 1/(1-\alpha)$ ; (b) if an equilibrium BGP exists it is unique; (c) the economy

<sup>&</sup>lt;sup>10</sup>This assumption is needed for (6) to have a non-zero solution. Given w(z), the function  $\pi$  is strictly convex in z with an elasticity of  $1/(1-\alpha)$ . Given  $z^E$ , the elasticity of  $\phi$  wrt to z' is 1+v; the condition  $v>\alpha/(1-\alpha)$  ensures that the latter is 'more convex' which is needed for an interior solution.

exhibits sustained growth, i.e. G > 1 if  $\beta > v_0 (1+v)/(1+v+vv_0\gamma)$ ; (d) if either condition in (a) holds and initially the economy is populated by homogeneous entrepreneurs, then the only equilibrium is the BGP; other non-explosive fluctuations in  $G_t$  are ruled out; (e) if either  $\gamma > 1 - \alpha/[(1-\alpha)v]$  or  $\gamma = 0$ , then, any equilibrium starting with initial distribution with bounded support will asymptotically converge to a homogeneous firms BGP; if  $\gamma = 0$  the convergence is after just one period.

As advanced previously, part (a) indicates for a BGP to exist, it might be necessary a high value for curvature parameter v. Also, part (e) suggests provides conditions upon which heterogeneity is not be sustained in a closed economy. Most obviously, if  $\gamma = 0$ , pre-existing heterogeneity disappears after one period; more interestingly, along the lines of Lemma 1, if  $\gamma > 1-\alpha/\left[(1-\alpha)v\right]$  the economy exhibits dispersion-induced homogeneity: it converges to a pool of homogeneous entrepreneurs because the top end of the distribution reproduces at a faster pace than the lower end; in the limit, all the remaining entrepreneurs would be the offsprings of the initially highest skilled entrepreneur(s).

For the remainder of the analysis, I will assume the sufficient conditions in Proposition 1, and use the homogeneous skills closed economy BGP in two ways: (a) as the initial conditions for both home and foreign at the time of openness the home country opens up, and (b) as the benchmark to assess the gains from openness.

### 3.3 The diffusion of foreign know-how in open economies

Consider now a country that freely allows old foreign entrepreneurs to set up firms and hire domestic labor.<sup>11</sup> I use, respectively, the names "home" and "foreign", and the indexes h and f, for the host country and for the rest of the world. I assume that initially both home and foreign are in the BGP as described in Proposition 1 and that at time t = 0 openness takes place unexpectedly and permanently. In addition, I assume that home is less developed, i.e. at time t = 0,  $Z_h < Z_f$ . I also assume that home is small, i.e. it does not affect the equilibrium path of the foreign country.

The entry of foreign entrepreneurs impacts the accumulation of skills at home in three ways. First, foreign firms enhance the exposure to ideas of the domestic young directly working for them. Second, foreign firms have spillovers on the country's level of  $Z^O$  which benefits all the local young, including those working for domestic firms. While these two effects are positive a third one is detrimental: foreign entrepreneurs bid up the cost of labor  $w_{t+1}(\cdot)$  in the country for all future periods t.

In each period, a free entry condition endogenously determines the mass of foreign know-how in the country. Foreign entrepreneurs enter until they are indifferent between operating at home or remaining in the foreign country. Since in the foreign country they earn  $\pi_f = (1 - \alpha) Z_f$  and there are no mobility frictions, their indifference between home and foreign can only happen when the (effective) cost of labor is the same in both countries.<sup>12</sup> Therefore, with openness, the domestic

<sup>&</sup>lt;sup>11</sup>I will assume that individuals from the home country cannot move. This is without loss of generality for workers and old entrepreneurs, since, in equilibrium they will be indifferent between moving to foreign or remaining in home. However, ruling out the possibility for domestic young potential entrepreneurs to move and "grow up" in the developed country is crucial. I will discuss further below the factual and analytical relevance of this assumption.

<sup>&</sup>lt;sup>12</sup>The key is that the effective unit of labor is the same in the two countries. For clarity, we assume that both countries have the same ratio of effective-to-physical units of labor. In the language of Burstein and Monge-Naranjo (2009), I am abstracting from differences in country embedded productivities. Adding those differences would add extra notation but no substance to the results in this paper.

market-clearing wages for foreign firms must be

$$w_h = w_f = \alpha Z_f. \tag{11}$$

Facing the same effective wages, each foreign firm hires the same number of labor units,  $n_f^* = 1$ , as if they had remained in the foreign country.<sup>13</sup>

Advance towards the analysis of a BGP in which, necessarily, the know-how levels of both home and foreign grow at a rate G > 1, it is convenient to consider the ratios  $R \equiv z/Z_f$ , i.e. the knowledge z of each domestic entrepreneur relative to the knowledge  $Z_f$  of the foreign entrepreneur. Likewise,  $R^O \equiv Z^O/Z_f$  denotes the external exposure to ideas in the home country relative to that in the foreign country.<sup>14</sup>

Openness to foreign skills can give rise to an interesting vintage structure for the population of domestic firms. To see this, Table 1 illustrates the diffusion of foreign skills into a country that opens up at time t=0. Denote by j=0,1,2,... the different generations of domestic offspring of foreign entrepreneurs. That is, j=0 indicates the foreign firm itself, j=1 indicates the generation of domestic entrepreneurs that directly worked for a foreign firm when young; j=2 are the domestic entrepreneurs that worked for a foreign-trained domestic entrepreneur, j+1 those who were trained by a member of generation j, etc. The ratio  $R_j^t$  indicates the skills, relative to skills in the foreign country,  $Z_f^t$ , of a generation j of domestic firms at time t.

For every period  $t \geq 0$ , a mass of foreign firms enters, each one carrying skills  $Z_f^t$ , the same skills as those firms that remained in the developed. Then, by definition,  $R_0^t = 1$  and since in each period  $w_h = w_f$ , each of the foreign firms hires  $n_f^t = 1$  domestic workers. Each of these domestic young individuals is exposed to ideas in the ratio  $R_0^{E,t} = (1)^{\gamma} \left(R^{O,t}\right)^{1-\gamma} \leq 1$  and acquire know-how in the ratio  $R_1^{t+1}$ . In the subsequent period t+1, they will expose each of their own  $n_1^{t+1}$  young workers to ideas in the ratio  $R_2^{E,t+1} = \left(R_1^{t+1}\right)^{\gamma} \left(R^{O,t+1}\right)^{1-\gamma}$ , which will form skills in the ratio  $R_2^{t+2}$ . Each of those will go on in period t+2 to spawn  $n_2^{t+2}$  entrepreneurs with skills  $R_2^{t+3}$  that will be active at t+3. And so on, as indicated by the diagonal arrows in Table 1. Clearly, with the passage of every period, an additional 'vintage' of domestic firms is added.

period\generation	0	1	2	3		j		$\infty$
0	1	=	-	-		-		=
1	1	$R_1^1$	-	-		-		-
2	1	$R_1^2$	$R_2^2$	-		-		-
3	1	$R_1^3$	$R_2^3$	$R_3^3$	• • •	-	• • •	-
:	:	÷	:	÷	:	÷	÷	:
t	1 🔍	$R_1^t \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$R_2^t$	$R_3^t \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$		$R_j^t \setminus$	• • •	-
<b>:</b>	:	:	:	:	÷	:	÷	÷
$-\infty$	1	$R_1^{\infty} \to$	$R_2^{\infty} \to$	$R_3^{\infty}$	• • •	$R_j^{\infty}$		$R_{\infty}^{\infty}$

**Table 1:** Relative skills  $R_i^t$  of the j-th generation of a foreign entrepreneur at period  $t \geq 0$ .

 $<sup>^{13}</sup>$ Recall that conceptually, this equality is in terms of effective units of labor. Therefore, the model can easily accommodate cross-country differences in workers earnings by introducing differences in effective units between workers of different countries. Indeed, if workers in the home country have fewer units of effective labor per unit of physical labor, foreign firms will be larger than the domestic firms in the foreign country. Moreover, since  $Z_f > Z_h$ , foreign firms are also larger than domestic firms at home.

 $<sup>^{14}</sup>Z_f^O = Z_f$  because the foreign country is in a BGP.

Asymptotically there can be an infinite number of vintages of domestic firms that have, directly or indirectly, built up their relative skills with foreign ideas.<sup>15</sup> In a steady or balanced growth path, all the ratios  $R_j^t$  –and the average  $R^{O,t}$  – would have converged to an array of constants  $\left\{R_j^{\infty}\right\}_{j=0}^{\infty}$ , and the absolute skills  $Z_j^t$  of all vintages grow over time at the gross rate G. As indicated by the horizontal arrows in the table, in the BGP, each vintage j exposes their workers to ideas in the constant ratio  $R_j^E = (R_j)^{\gamma} \left(R^O\right)^{1-\gamma}$ , these workers form skills in the ratio  $R_{j+1} \leq R_j$ .

For simplicity, and without the risk of confusion, I will ignore the superindex  $\infty$  when discussing the BGP.

#### 3.3.1 Balance Growth Paths

In a BGP of an open economy, a constant mass  $m_f \geq 0$  of foreign firms enter the country and a constant distribution of skill levels R (the relative gaps with respect to the foreign productivity  $Z_f$ ) characterize the population of domestic firms. Obviously, the external exposure inside the country relative to that in the foreign country,  $R^O = Z^O/Z_f$ , would also be constant along a BGP.

It is convenient to separate the more general case  $0 < \gamma < 1$  from the two special cases  $\gamma = 0$  and  $\gamma = 1$ .

 $0 < \gamma < 1$ : Partial Internalization with Externalities. When  $0 < \gamma < 1$ , there are also two BGP. First, 'full-convergence',  $1 = R^O = R_j$  all j, is a BGP. In such a BGP, the (net) entry of foreign firms is zero,  $m_f = 0$ , all domestic firms are homogeneous and identical to foreign firms in terms of skills, wages and learning same learning opportunities. If such BGP is reached, the equilibrium conditions are the same as in the closed economy BGP.

A second, 'interior' or 'partial convergence' BGP exists in which the skills and ideas circulating in the home country are inferior to those in the developed foreign countries, i.e.  $R^O = Z^O/Z_f < 1$ , and  $R_j \leq 1$  all j with the inequality strict for at least some  $j < \infty$ . To lay out the equilibrium conditions for this interior BGP, let  $w_j$  denote the wage that a domestic firm of vintage j must pay their workers. Because all young individuals are ex-ante identical and all have the option to work for any firm, whenever  $R_j < 1$  the wage  $w_j$  must be higher than  $w_f = \alpha Z_f$  to compensate for the inferior learning opportunities. Define  $d_j \geq 0$  to be the relative compensating differential in terms of  $Z_f$ , i.e.  $w_j = (\alpha + d_j) Z_f$ . In a BGP the compensating differentials must be an array of non-negative constants  $\{d_j\}_{j=0}^{\infty}$ . By construction,  $R_0 = 1$ ,  $d_0 = 0$ ,  $n_0 = 1$ , and  $R_0^E = (R^O)^{1-\gamma}$ . Since the aggregate mass of labor is normalized to 1, and  $n_f = 1$ , then  $m_0 = m_f$ .

An equilibrium BGP of an open economy is a domestic-to-foreign ratio of external exposure of ideas  $0 < R^O \le 1$ , and an array of non-negative employment shares, firm sizes, relative skills, compensating differentials and exposures to know-how  $\{m_j, n_j, R_j, d_j, R_j^E\}_{j=0}^{\infty}$  such that,  $\forall j = 0, 1, 2, ...$ 

(a) old entrepreneurs of vintage j maximize profits, hiring workers in the amount

$$n_j = \left[\frac{\alpha}{\alpha + d_j} R_j\right]^{\frac{1}{1 - \alpha}},\tag{12}$$

(b) the employment shares for each j are

$$m_j = m_0 \prod_{i=0}^{j} n_i,$$
 (13)

<sup>&</sup>lt;sup>15</sup>Once again, when the occupation choices are introduced in Section 4, the number of vintages may be finite.

(c) and the entry of foreign firms  $(m_f = m_0)$  insures that the country's labor market clears

$$m_0 = \frac{1}{\sum_{k=0}^{\infty} \prod_{j=0}^{k} n_j}.$$

$$(14)$$

(d) The home country's relative external exposure of ideas is the weighted mean of all the skills implemented

$$R^{O} = \left[\sum_{j=0}^{\infty} m_{j} \left(R_{j}\right)^{\rho}\right]^{\frac{1}{\rho}}; \tag{15}$$

(e) and for each firm of vintage j, the relative exposure to ideas is

$$R_j^E = (R_j)^{\gamma} \left( R^O \right)^{1-\gamma}. \tag{16}$$

(f) Given exposure  $R_{j-1}^E$  and future wages  $(\alpha + d_j) GZ_f$ , skill investments  $R_j$  of workers in vintage j-1 are optimal:

$$\beta \left( \frac{\alpha R_j}{\alpha + d_j} \right)^{\frac{\alpha}{1 - \alpha}} \left[ 1 + \frac{\alpha R_j^E}{\alpha + d_j} \frac{\gamma v v_0}{1 + v} \left( G \frac{R_{j+1}}{R_j^E} \right)^{1 + v} \right] = v_0 \left( G \frac{R_j}{R_{j-1}^E} \right)^v; \tag{17}$$

(g) and young individuals are indifferent to work in the different active firms  $j \geq 1$ 

$$d_{j} = \beta G \left[ \pi \left( R_{1}, \alpha + d_{1} \right) - \pi \left( R_{j+1}, \alpha + d_{j+1} \right) \right] + R_{j}^{E} \phi \left( G \frac{R_{j+1}}{R_{i}^{E}} \right) - \phi \left( G \frac{R_{1}}{R_{0}^{E}} \right), \tag{18}$$

where the functions  $\pi(\cdot, \cdot)$  and  $\phi(\cdot)$  are as defined above and are used here to shorten the last expression.

Using the parameter values discussed in Section 5, Figure 1 illustrates the behavior of the different vintages of domestic firms in an interior BGP with  $0 < \gamma < 1$ . Because  $R^O < 1$ , all the young domestic entrepreneurs are less exposed to productive ideas than their foreign counterparts. Even those working in foreign firms are only exposed to the ratio  $\left(R^O\right)^{1-\gamma} < 1$ . For those working in older vintages, the exposure to ideas decays consistently,  $R_j^E \geq R_{j+1}^E$ . As shown in the upper-left panel, this results in a formation of skills that consistently decays with the age of the vintage. The declining exposure to ideas also explain that compensating differentials  $d_j$  must increase with the vintage as shown in upper-right panel of the figure. Both, the declining skills  $R_j$  and the increasing wages  $\alpha + d_j$  explain the decline in the labor hired by each firm  $n_j$  and in the share  $m_j$  of the total labor force hired by the vintage j (bottom-left panel). In this example, the decline is so rapid that shares  $m_j$  are negligible for  $j \geq 5$ .

The bottom-right panel of the figure reports the difference between the earnings (relative to  $Z_f$ ) of someone who, having worked in vintage j when young, invests in skills and becomes an entrepreneur (i.e.  $v_j \equiv V_j/Z_f$ ) and the income  $\beta G(\alpha + d_j)$  of remaining a worker value in the same vintage j next period (and thus and not investing). The fact that  $v_j - \beta G(\alpha + d_j)$  becomes negative for higher j indicates that entrepreneurship choices will be binding, i.e. at least some of the old would want to remain workers. Before considering occupation choices in the analysis, it is convenient to consider the remaining parameter configurations.

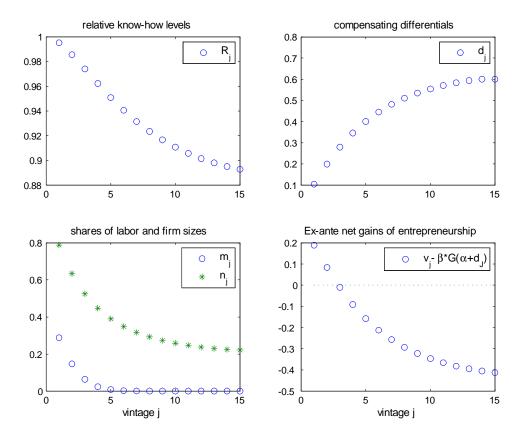


Figure 1: Interior BGP of an open economy (no occupation choices).

 $\gamma=1$ : Full internalization. In this case the diffusion of ideas is entirely within the firm,  $z^E=z$ . The outside set of ideas  $R^O$  is irrelevant (as is the value of  $\rho$ ). In particular, regardless of how productive are the domestic firms in the home country, workers in a foreign firm are exposed to exactly the same set of ideas as foreign workers in the foreign country. They will be able –and will find it optimal—to acquire the same level of skills as their foreign peers, i.e.  $R_0^{t+1}=1$ . In the next period, these "new" domestic entrepreneurs will expose their own workers to the same level of skills as the foreigners. Their workers will therefore accumulate the same level of skills as young foreigners, i.e.  $R_{j+1}^{t+1}=1$ , and so on. Because of this, in any period t, and all vintages t0 the mass of foreign firms t1. Then, to characterize the equilibrium, it suffices to keep track of t2 the mass of foreign firms t3 the total mass of new domestic firms t4 (i.e. the cumulative mass of all the different generations t3 of foreign-trained domestic entrepreneurs) and t4 the relative productivity, t5 of the old or pre-existing domestic firms at the time t5 when the country opened up.

As time passes by, the cumulative entry  $\sum_{\tau=0}^{t} m_f^{\tau}$  of foreign firms builds up the mass  $m_{\text{new}}^t$  of foreign-trained domestic entrepreneurs, a sector which will eventually overtake the entire labor force, i.e.  $m_{\text{new}}^t \to 1$ . In the BGP, these domestic entrepreneurs alone push the equilibrium domestic wage to the foreign level  $w_f$ , and foreign firms will no longer have gains of entering, i.e.  $m_f = 0$ .

 $\gamma=0$ : Only Externalities. This is the opposite case, as the diffusion of foreign ideas is only through spillovers on  $Z^O$ . Because  $z^E=Z^O$ , domestic and foreign firms both offer the same learning opportunities to their workers and must pay the same wages. Contrary to the case of  $\gamma=1$ , the productivity of domestic and foreign firms can exhibit persistent gaps, i.e. R<1, if

in equilibrium there are persistent differences in the set of ideas circulating in the home and in the foreign countries, i.e.  $R^O < 1$ . In such interior BGP, the inflow of foreign skills reduces the marginal return of investing in skills in the same proportion as the embedded inflow of foreign ideas reduces the marginal cost of investing skills. Interestingly, the interior BGP is unique and globally stable. Then, the only way that home can fully catch up  $(R = R^O = 1)$  is if the country is already there.

To see this, when  $\gamma=0$  we can write the transition function for  $R^t\equiv Z_h^t/Z_f^t$  in closed form.<sup>16</sup> Given  $Z_h^t$  and  $w^t=\alpha Z_f^t$ , each domestic entrepreneur hires  $n_h^t=(R^t)^{\frac{1}{1-\alpha}}$  units of labor. Since each foreign firm hires  $n_f^t=1$ , the clearing of the domestic labor market requires  $m_f^t=1-(R^t)^{\frac{1}{1-\alpha}}$ . The shares of labor hired by domestic and foreign firms are, respectively,  $1-m_f^t$  and  $m_f^t$ , implying that the relative exposure to ideas for youth at home is

$$R^{E,t} = R^{O,t} = \left[1 + \left(R^t\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^t\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}.$$
 (19)

Openness always improves the domestic exposure to ideas, i.e.  $R^{E,t} > R^t$ , because  $R^{E,t}$  is an average of 1 and  $R^t \le 1$ . However,  $R^{E,t}$  might is not be monotone increasing in  $R^t$ . There are two countervailing forces. On the one hand, a higher  $R^t$  increases  $R^{E,t}$  because domestic firms are a better source of ideas. On the other hand, a higher ratio  $R^t$  reduces the entry of foreign  $m_f^t$  and the country's exposure to to foreign ideas. When  $\rho \le -1/(1-\alpha)$ , the first effect dominates and  $R^{E,t}$  is always increasing in  $R^t$  because of a strong complementarity between the domestic and foreign sources of ideas. However, if  $\rho > -1/(1-\alpha)$ , the negative effect dominates for low values of  $R^t$  and the relative exposure  $R^{E,t}$  increases with a higher  $R^t$ .

Openness does not lead to full convergence; that is, even if  $R^{E,t} > R^t$ , the relative skills of domestic firms will remain below those of foreign firms, i.e.  $R^{t+1} < 1$ , because they build up their skills on the basis of interior ideas,  $R^{E,t} < 1$ , but have to pay the same wages  $w_{t+1} = \alpha G Z_f^t$  as foreign firms. To see this, solve for the optimal accumulation of skills (17) for  $\gamma = 0$  and use (19), which leads to

$$R^{t+1} = (R^{E,t})^{\mu} = \left[1 + (R^t)^{\rho + \frac{1}{1-\alpha}} - (R^t)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}}, \tag{20}$$

where  $\mu \equiv v/\left[v-\alpha/\left(1-\alpha\right)\right] > 1$ . Thus,  $R^{t+1}$ , the next period's relative skills of domestic entrepreneurs is a strictly convex function of the current period relative exposure  $R^{E,t}$ . An obvious fixed point in this mapping is when  $R = R^E = 1$ . But, as long as  $-\infty < \rho < +\infty$ , another unique and (stable) interior fixed point exists  $R^{\text{int}}$  in which  $0 < R^{\text{int}} \le R^{O,\text{int}} \le R^{E,\text{int}} < 1$ . This is (barely) shown by Figure 2, which is constructed with the parameter values discussed in Section 5. In one limit, when  $\rho = -\infty$ , the interior BGP converges to  $R^{\text{int}} = 0$  and all firms in the country will be controlled by foreign skills. On the other limit, when,  $\rho = +\infty$  the interior BGP collapses to R = 1, and, after just one period after openness the home country will fully catch up with the foreign countries.

Appendix A contains the proof of the following result:

**Proposition 2** (BGP open economy) Assume that the parameter assumptions of Proposition 1 hold. Then: (a) Full convergence, i.e.  $R^O = 1$  is always a BGP equilibrium; (b) if either  $\gamma = 1$  or  $\rho = \infty$ , then full convergence is the unique BGP and the country converges to it from any

The vintage structure is not needed as  $R_j = R$  and  $d_j = 0$  for all  $j \ge 1$ .

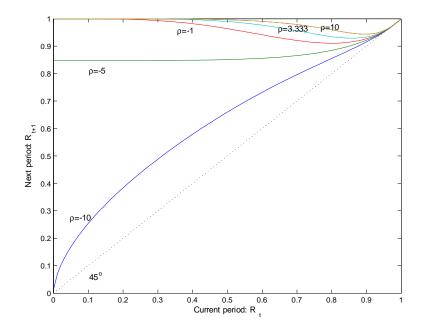


Figure 2: Transition function of an open economy,  $\gamma = 0$ .

initial condition; (c) if  $\gamma = 0$ , and  $\rho < \infty$ , then there exists a unique interior equilibrium, i.e.  $0 < R^{O,int} < 1$  and the country converges to it from any initial condition  $R^0 = Z_b^0/Z_f^0$ .

The equivalent of part (c) for  $0 < \gamma < 1$  is harder to prove analytically because of the multiplicity of dimensions and the potentially non-monotonicity of  $R^O$  as a function of  $\{R_j\}$ . However, it is straightforward to examine numerically. Indeed, with the parameter values used in Section 5, uniqueness and global stability of the interior BGP was routinely verified when  $\gamma > \alpha/[v(1-\alpha)]$ , as suggested by Lemma 1.

Having the case of  $\gamma = 0$  in closed-form yields two simple but useful analytical results about the gains from openness. Both of these results highlight a strong regressivity in the gains of openness. The first one is that after openness, countries that lag behind could surpass other developing countries that started ahead. Denote by  $R_i^t$ , the relative know-how of domestic firms at time t of a country that opened up at t = 0 with initial relative productivity  $R_i$ . Then:

Corollary 1 (Leapfrogging 1) Assume that  $\gamma = 0$  and that  $\rho > -1/(1 - \alpha)$ . Then there are two initial levels  $R_1 < R_2$  such that  $R_1^t > R_2^t$  for  $t \ge 1$ .

When  $\rho > -1/(1-\alpha)$ , the relative exposure  $R^E$  is initially decreasing with respect to the relative productivity of domestic firms R. This is because, at low levels of R, the complementary between domestic and foreign sources of ideas is not strong enough to outdo the higher entry of foreign ideas. Then, taking two small countries that under closeness lag behind the developed world, if both of them open, the country that is further behind will receive more entry, and its youth will be exposed to more ideas. Because of this, the domestic firms of that country will be managed with more knowledge in the next period. Interestingly, the form of the transition function implies that after a period, countries leave the decreasing region and forever after remain in the increasing portion of the transition. Then, even if both countries will eventually converge to same  $R^{\text{int}}$ , during the entire transitions, the initially behind country will stay ahead.

Aggregation in the case of  $\gamma = 0$  is straightforward. In the closed economy, aggregate geographic and national output are equal and given by  $Y_{\text{closed}}^t = Z_h^t$ . Subtracting the costs of learning,

aggregate consumption is  $C_{\text{closed}}^t = Z_h^t \left[ 1 - \phi \left( G \right) \right]$ . For an open economy, free entry of foreign firms implies that domestic (geographic) output will also be equal to  $Y_{\text{open}}^{D,t} = Z_f^t$ , and subtracting foreign profits, national income is  $Y_{\text{open}}^{N,t} = Z_f^t \left[ \alpha + \left( 1 - \alpha \right) R_t^{\frac{1}{1-\alpha}} \right]$ , which is obviously increasing in  $R_t$  (less of the output goes away as to foreign profits when the local entrepreneurs are more skillful). After some easy manipulations, it can be shown that national consumption is

$$C_{\text{open}}^{t} = Z_{f}^{t} [\alpha + (1 - \alpha) R_{t}^{\frac{1}{1 - \alpha}} - \phi(G) (R_{t}^{E})^{\mu + (\mu - 1)v}].$$

Consumption is a decreasing function of  $R_t^E$  because it increases the investment in skills of the young generation. Define the *steady state* welfare gains of openness as  $C_{\text{open}}^{\text{int}}/C_{\text{closed}}^0 - 1.^{17}$  Using the definition of  $\mu$  and simplifying,

$$\frac{C_{\text{open}}^{\text{int}}}{C_{\text{closed}}^{0}} = \frac{1}{R^{0}} \frac{\alpha + \left[1 - \alpha - \phi\left(G\right)\right] \left(R^{\text{int}}\right)^{\frac{1}{1-\alpha}}}{1 - \phi\left(G\right)},$$

immediately leading to the following result:

Corollary 2 (Steady state gains from openness) The steady state output and welfare gains of openness are strictly decreasing in  $R^0$ ; if  $R_L \equiv [\alpha + (1 - \alpha - \phi(G))(R^{int})^{\frac{1}{1-\alpha}}]/[1 - \phi(G)] < 1$ , then countries with  $R_L < R^0 < 1$  have negative welfare gains when they open.

How can a country lose with openness when it brings the superior knowledge from abroad? Because the *future* inflow of foreign skills reduces the incentives of each individual in the current generation to build up skills. Collectively, this reduces the value of the public good  $Z^O$ , and reduces the ability of forming skills for everyone in the future generation.

To see this more clearly, consider the extreme case in which  $z^E = Z_h$ , i.e. the domestic young can only learn from the domestic old. This case is in the spirit of Stokey (1991) where openness changes the relative price of factors of production –in favor of labor, the factor that does not require investment– but does not allow for international diffusion of knowledge. In this case, the transition function boils down to  $R_{t+1} = R_t^{\mu}$  and the stable BGP is  $R^{\text{int}} = 0$ . Then, whenever the initial  $R^0$  lays below 1, openness leads the country to destroy the platform of ideas upon each generation built up their skills. In the limit, the country ends up fully specialized in providing labor (to foreign firms). Aggregate consumption and national income both would equal  $\alpha Z_f^t$ . Steady state gains of openness are positive only if  $R^0 < \alpha/[1 - \phi(G)]$ .

The essence of these results, as argued in sections 4 and 5, extend to the general model with  $0 < \gamma < 1$ , with occupation choices and transitional dynamics. There, a central issue will be the quantitative magnitude of the gains (or losses) of openness.

## 4 Occupation Choices and the Diffusion of Know-How

Entrepreneurship choices have a prominent presence in the development literature (e.g. Banerjee and Newman 1993). Sorting individuals between managerial and labor occupations can enhance the static gains of openness as shown by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully by Eeckhout and Jovanovic (2009). In this section I will argue that occupation choices can also determine whether –and how quickly– a developing

<sup>&</sup>lt;sup>17</sup>In Section 5 I account for transitional dynamics when computing the gains of openness.

country can catch up with the rest of the world. Specifically, I will show that occupation choices: (a) can change the form of the BGPs; (b) can push an open economy away from the interior BGP and instead to fully catch up; and (c) can accelerate the convergence.

In the model, an old person carrying a skill level z would only become an active entrepreneur if his rents  $\pi[z, w(z)]$  are above the maximum wage as a worker, i.e. only if

$$\pi \left[ z, w_t \left( z \right) \right] \ge \sup_{\zeta \in \text{support}(t)} w_t \left( \zeta \right), \tag{21}$$

where 'support' refers to the entire set of entrepreneurial knowledge –domestic or foreign– active in the country.

The option of choosing occupation when old can change the investment in skills for a young person. For a given exposure to ideas  $z^E$ , a young person would only invest in skills if:

$$V\left[z^{E}, w_{t+1}\left(\cdot\right)\right] \ge \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}\left(\zeta\right). \tag{22}$$

This lower bound in the career value of a job  $V[\cdot,\cdot]$  can reduce the equilibrium gap between the wages paid by active entrepreneurs with different skills. Specifically, consider two entrepreneurs with skill levels  $z_0 < z_1$ . If the two of them fall below a certain threshold  $z_t^*$ , they will both pay the same wage; if the two fall above the threshold, the wage difference will be given by (8) of the previous section, reflecting the difference in the learning opportunities of the two jobs. Finally, if the two skill levels fall on different sides of the threshold, i.e.  $z_0 < z_t^* < z_1$ , the two wages paid satisfy:

$$w_{t}(z_{1}) = w_{t}(z_{0}) + \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}(\zeta) - V\left[z_{1}^{E}, w_{t+1}(\cdot)\right] < w_{t}(z_{0}) = w_{t}(z_{t}^{*}).$$

Obviously,  $w_t(\cdot)$  is flat up to the threshold  $z_t^*$ , after which it becomes strictly decreasing.

Identical results as Lemma 1 and Proposition 1 of the Section 3 hold even with a flat region of  $w_t(\cdot)$ . More interestingly, when  $\gamma > 1 - \alpha/[(1 - \alpha)v]$ , convergence to the BGP with homogeneous entrepreneurs can be even faster as the lower tail of the skill distribution is being eliminated each period; the only ones to reproduce are the entrepreneurs in the higher end.

Before analyzing the impact on an open economy, notice that occupation choices can also change the BGP in a close economy.

**Lemma 2** (Closed economy BGP, occupation choices) Under the same parameter assumptions as in Proposition 1, there exists a unique BGP in a closed economy with occupation choices. If the G in the BGP without occupation choices satisfies  $\beta(1-2\alpha) > v_0G^v/(1+v)$ , then it is also the BGP with occupation choices. If not, the unique BGP is described by a fraction  $\omega$  of young individuals who invest in skills and an intergenerational growth rate G of skills, such that (i) young individuals are ex-ante indifferent between the two occupations:

$$\beta \left[ \frac{2 - \omega}{\omega} \right]^{\alpha} \left[ \frac{2(1 - \alpha) - \omega}{2 - \omega} \right] = \frac{v_0}{1 + v} G^v;$$

and (ii) those who invest do so optimally, i.e. G is the lower root of

$$\beta \left[ \frac{2 - \omega}{\omega} \right]^{\alpha} \left[ 1 + \frac{\gamma v v_0}{(1 + v)} G^{1 + v} \right] = v_0 G^v.$$

For the rest of the analysis, however, I will focus on the case where  $\beta (1 - 2\alpha) > v_0 G^v / (1 + v)$ . In the close economy BGP all the young become entrepreneurs when old.<sup>18</sup>

#### 4.1 Open Economies

It is convenient to keep separate the different cases for  $\gamma$ :

 $\gamma=1$ : Full internalization. As before, full convergence is the only BGP for an open economy. Those who work for a foreign firm will mature next period into a domestic entrepreneur with exactly the same level of know-how as the contemporaneous foreign entrepreneurs. Overtime, this new group of domestic entrepreneurs fully takes over the domestic labor of the country and foreign firms will cease to enter.

Occupation choices can accelerate the convergence to R=1. To illustrate this, consider a country initially endowed with domestic entrepreneurs with very low skills,  $R < \alpha/(1-\alpha)$ . These initial old entrepreneurs would rather supply their labor, and all the young will work for foreign firms. Because of this, the country will attain R=1 the next period after openness. Notice that without occupation choices the convergence would only be asymptotic as the initial old would have to remain active and would reproduce over time.

But even if the old generation of domestic entrepreneurs is skilled enough to remain active,  $R > \alpha/(1-\alpha)$ , occupation choices can accelerate the convergence to the BGP. For instance, their own workers may not find it optimal to invest in skills due to their relative inferior exposure to knowledge. If so, in the next period they would be workers, and the economy will converge after two periods of openness. It is possible that convergence requires an integer n of generations of progeny of the pre-existing domestic entrepreneurs until the n-th one finds it optimal not to invest.

In any case, with occupation choices convergence is in finite time, not just asymptotic.

 $\gamma = 0$ : Only Externalities. Absent occupation choices, the interior BGP was globally stable. With occupation choices, however, the interior BGP may no longer exist and an open economy necessarily converges to R = 1.

There are two possibilities. The first is driven by the occupation choices of the old: with relative skills  $R^{\text{int}}$  the old may be better off as workers, not as entrepreneurs. This happens when

$$R^{\text{int}} < \left[\alpha/\left(1 - \alpha\right)\right]^{1 - \alpha}. \tag{23}$$

because the domestic profits  $\pi_h = \theta Z_h^{\frac{1}{1-\alpha}} [w_f]^{\frac{-\alpha}{1-\alpha}}$  fall short of the wages  $w_f = \alpha Z_f$ . The second possibility is that the young, being exposed to  $R^{O,\text{int}} = \left[1 + \left(R^{\text{int}}\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^{\text{int}}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}$ , could be better-off not investing and remaining workers. To see this, as a worker next period, the young foresees discounted earnings of  $w^{t+1} = \beta G \alpha Z_f^t$ ; should he opt to be an entrepreneur his optimal acquisition of skills is  $Z_h^{t+1} = Z_f^t G \left(R^{O,\text{int}}\right)^{\mu}$ , and his net discounted payoff would be equal to  $Z_f \left\{\beta \left(1-\alpha\right) G \left(R^{O,\text{int}}\right)^{\frac{\mu}{1-\alpha}} - R^{O,\text{int}} \phi \left[G \left(R^{O,\text{int}}\right)^{\mu-1}\right]\right\}$ . After some basic simplifications, it can be shown that a young person would not become an entrepreneur when

$$R^{O,\text{int}} < \left[ \frac{\beta G \alpha}{\beta (1 - \alpha) G - \phi (G)} \right]^{\frac{1 - \alpha}{\mu}}, \tag{24}$$

<sup>&</sup>lt;sup>18</sup>The other case is a knife-edge for open economies. As it will become evident in this section, if  $\beta (1-2\alpha) \le v_0 G^v/(1+v)$ , upon openness, the country will fully catch up with developed countries after one period (if  $R < \alpha/(1-\alpha)$  as all the old domestic entrepreneurs choose to be workers) or two periods (if  $\alpha/(1-\alpha) < R < 1$  as the current young do not invest).

i.e. if his exposition to ideas is too low relative to that of the future foreign competition.

If either of these two conditions hold, an open country will always catch up with the developed countries a finite number of periods after openness. This is because, regardless of initial conditions,  $R^t$  eventually gets near  $R^{\text{int}}$ , and, at that point, after one or two periods, the country would jump to R = 1.

Even if neither (23) nor (24) rule out the interior BGP, depending on initial conditions, occupation choices can lead an open country to R=1. If in any period  $R^t$  falls below the threshold  $\left[\alpha/\left(1-\alpha\right)\right]^{1-\alpha}$ , then all the old would opt to remain workers, clearing the way for the young to be exposed to only foreign ideas, i.e.  $R^{O,t}=1$ , leading to  $R^{t+1}=1$ . Likewise, the occupation choice of the young could trigger the convergence; if  $\left[\alpha/\left(1-\alpha\right)\right]^{1-\alpha} < R_t < R^{O,t} < \left(\beta G\alpha/\left[\beta\left(1-\alpha\right)G-\phi\left(G\right)\right]\right)^{\frac{1-\alpha}{\mu}}$ , the current remains active but the young do not invest,  $R^{t+1}=0$ ; then, all the entrepreneurial skills at t+1 will be foreign,  $R^{O,t+1}=1$ , and the young at t+2 will invest at the rate  $R^{t+2}=1$ , and all generations thereafter will remain there.

In sum, more than just accelerating the time to convergence, when  $\gamma = 0$  occupation choices can completely change the *direction* of convergence in favor of full catch-up.

 $0 < \gamma < 1$ : Partial Internalization with Externalities. Once occupation choices are present, it is possible that only a finite number  $J < \infty$  of vintages of domestic firms can remain active. This was suggested by Figure 1, where young entrepreneurs in older vintages j would be better off remaining workers. Therefore, in equilibrium there is a last vintage in which the young workers in the last vintage j = J do not invest in skills and instead returns to the same vintage j = J in period t + 1 as an old worker.

Denote by  $d_J$  the compensating differential (in units of  $Z_f$ ) received by workers of the last vintage J. Since entrepreneurs do not care who is providing the labor, both young and old receive wages equal to  $w_J = (\alpha + d_J) Z_f$ . For a young person to be indifferent between being a worker in both periods of life, the implied lifetime earnings  $(\alpha + d_J)(1 + \beta G)$  must be equal to the lifetime earnings attained from any other option, in particular j = 0, the foreign firm. Equating the net-present values from these two options, the compensating differential  $d_J$  must be equal to

$$d_{J} = \frac{\beta G \left[\pi \left(R_{1}, \alpha + d_{1}\right) - \alpha\right] - R_{0}^{E} \phi \left(G \frac{R_{1}}{R_{0}^{E}}\right)}{1 + \beta G}.$$
 (25)

In all the other vintages, j = 0, ..., J - 1,  $d_j$  behaves as in the previous section, with labor fully provided by young individuals who invest in skills and become entrepreneurs when old. Since there is no population growth, it has to be the case that half of the workers in the last vintage J are young and half old.

Therefore, an interior BGP equilibrium (i.e.  $0 < R^O < 1$ ) of an open economy with occupation choices is defined as a  $J \in \mathbb{N}$  (possibly  $\infty$  if occupation choices do not bind) and an array  $\{m_j, n_j, R_j, d_j, R_j^E\}_{j=0}^J$  that satisfies conditions (a)-(b) and (d)-(g) of the previous section, but with the following modifications: (i) the summations run from j=0 to j=J; (ii)  $d_J$  is given by (25); and (iii) while condition (c) holds, the absolute mass of foreign firms is  $m_f = m_0 (1 + n_J/2)$  since the total labor force includes  $n_J/2 > 0$  old persons supplying labor for the last vintage of domestic firms.

If occupation choices not bind, a number  $J < \infty$  of vintages is determined by the following conditions: (1) workers in vintage J-1 find it optimal to invest at the rate  $R_J$  instead of remaining a worker:

$$\beta G\left[\pi\left(R_{J},\alpha+d_{J}\right)-\left(\alpha+d_{J}\right)\right]-R_{J-1}^{E}\phi\left(G\frac{R_{J}}{R_{J-1}^{E}}\right)>0;$$

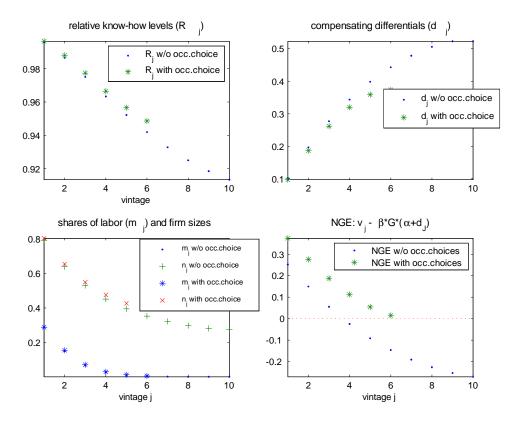


Figure 3: BGP open economy, with occupation choices; only 6 vintages of domestic firms

and (2) workers in vintage  $R_J$  find it optimal to remain workers, given that, as entrepreneurs they would not spawn entrepreneurs, i.e.

$$\max_{R} \left\{ \beta G \left[ \pi \left( R, \alpha + d_{J} \right) - \left( \alpha + d_{J} \right) \right] - R_{J}^{E} \phi \left( G \frac{R}{R_{J}^{E}} \right) \right\} < 0.$$

This vintage structure is similar to that in Chari and Hopenhayn (1991), except for two crucial additional aspects. First, the levels  $R_j$  of each vintage  $j \ge 1$  are endogenous. Second, the level of the externality  $R^O$  is also determined endogenously.

Comparing economies with and without occupation choices, it is evident that occupation choices have two different positive effects on the formation of domestic know-how. First, a 'selection effect', as the older, less productive vintages are no longer active. Their labor must be re-allocated to the younger, more productive vintages. This reallocation pushes the economy to a higher ratio  $R^O$  of ideas. Second, there is an 'investment effect': the young working in any of the active vintages j = 0, ..., J - 1 invest more because of the lower compensating differentials  $d_j$  that they would would have to pay, given the better career options in vintages  $j \geq J$ .

With the parameter values of Section 5, Figure 3 compares the interior BGPs of economies with occupation choices (dots) and without them (stars). As shown by the lower-right panel, only six vintages of domestic firms would remain active. As shown by the upper-left panel, the relative productivity  $R_j$  of the active vintages are higher than without occupation choices and, as shown by the upper-right panel, the compensating differentials  $d_j$  are lower. With this, it is easy to explain why the labor units  $n_j$  and the shares of labor  $m_j$  hired by the six surviving vintages are higher with occupation choices (lower-left panel).

As in the case of  $\gamma = 0$ , the impact of occupation choices goes beyond changing the structure of interior BGPs. First, they can remove the interior BGP altogether, and regardless of initial conditions, openness would necessarily lead a country to fully catch up with developed countries. Second, even if the interior BGP persists, occupation choices can drastically change the dynamics of an open country because, depending on initial conditions, the country would move towards full convergence instead of the interior BGP.

Two simple corollaries summarize the effect of occupation choices on the diffusion of foreign entrepreneurial know-how in open economies.

Corollary 3 (Destruction for convergence) Open countries only catch up with developed countries if pre-existent, inferior domestic skills stop being reproduced at some point. If so, such destruction is abrupt (in one period) if  $\gamma = 0$ , but could be gradual if  $\gamma > 0$ .

Just because of occupation choices, the economy may no longer converge to the interior BGP. However, an open country only catches up if, along the way, the pre-existent, less productive domestic sector is entirely replaced but a new domestic sector with the same productivity as foreign firms.

Corollary 4 (Leapfrogging 2) Assume the interior BGP exists (i.e.  $R^{int} > [\alpha/(1-\alpha)]^{1-\alpha}$  and  $R^{O,int} > \{\beta G\alpha/[\beta(1-\alpha)G-\phi(G)]\}^{\frac{1-\alpha}{\mu}})$ . Then, there are two initial levels  $R_1^0 < R_2^0$  such that, upon openness at t = 0,  $\lim_{t\to\infty} R_2^t = R^{int} < \lim_{t\to\infty} R_1^t = 1$ .

Simply put, after openness the more backward may end up will fully catching up with developed countries while the more advanced one will remain forever behind. This form of leapfrogging is in BGPs and is very different from the one in the previous section, which was in the transition towards the same interior BGP. Since the conditions for the two are very different, either one could hold without the other. However, both forms tend to enhance the gains of openness more for the more backward countries.

### 5 Quantitative Analysis

This section provides a basic quantitative assessment of the aggregate gains that a developing attains from being open to foreign firms from more developed countries. To this end, I extend the basic model to connect with some key observations about entrepreneurial activity. The extensions also allows me to use parameter values that are standard in the literature without provoking unrealistic reactions from occupation choices. Moreover, without adding or subtracting analytical results, the extended model provides a more accurate depiction of the diffusion of entrepreneurial skills, as it is restricted to a small fraction of young individuals who directly interact with the decision-makers at the top of the firms.

After briefly setting up the extended model, I discuss the parameter values used, and then I illustrate the dynamics of a country after it opens up. Then, I assess the welfare gains for different initial conditions and alternative values for the internalization and diffusion parameters.

### 5.1 A Model with Middle-Managers and Workers

Consumption goods are produced with entrepreneurial (top management) skills (z), mid-management services (n), and labor services (l):

$$y = z n^{\alpha} l^{\lambda}.$$

Each young cohort is composed of two groups: a fraction  $\omega < 1/2$  of potential managers and a fraction  $1-\omega$  of perennial workers. Perennial workers provide labor services when young and when old and do not accumulate skills. On the contrary, potential managers, in either period can be workers or middle-managers. Moreover, when young they can invest in skills and become entrepreneurs (top-managers) when old. As in the basic model, their exposure to ideas  $z^E$  is determined by both, the skills z of the top-manager for whom the young person works and the average  $Z^O$  implemented in the country similar to the basic model.<sup>19</sup>

When all the  $\omega$  entrepreneurs have skills  $Z_t$  and the total supply of managerial skills is  $\omega Z_t$ ; the aggregate supply of labor is  $2(1-\omega)$  and the aggregate supply of mid-management services is  $\omega$ . Under the parameter values used (see the discussion below), in the closed economy BGP all the potential young managers are middle-managers, invest in skills, and become active entrepreneurs when old. Production teams are composed of one top manager, one middle manager and  $\varrho \equiv 2(1-\omega)/\omega > 1$  workers. The income of workers  $(w_t)$ , mid-managers  $(w_t^m)$  and entrepreneurs  $(\pi_t)$  are, respectively,  $w_t = \lambda \varrho^{\lambda-1} Z_t$ ,  $w_t^m = \alpha \varrho^{\lambda} Z_t$ , and  $\pi_t = (1-\alpha-\lambda) \varrho^{\lambda} Z_t$ . It is straightforward to verify that if  $\lambda/\varrho < \alpha < (1-\lambda)/2$  ex-post occupation choices are satisfied as an old person is better-off being a top-manager than a mid-manager or a worker and a young manager is better-off than a worker.

As detailed in Appendix B, along the closed economy BGP, the gross growth rate of skills G is given by

$$\beta \left[ \varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v} \right] = v_0 G^v,$$

the same as expression (10) except for the term  $\varrho^{\lambda}$ . Since  $\varrho^{\lambda} > 1$ , top-managers have more workers under their control. Finally, for an equilibrium BGP it is required that potential managers opt to invest in skills instead of remaining mid-managers when old, i.e.  $w_t^m + \beta w_{t+1}^m < w_t^m + \beta \pi_{t+1} - Z_t \phi(G)$ . This inequality boils down to  $\phi(G) < \beta(1 - 2\alpha - \lambda) \varrho^{\lambda}G$ . Thus, under conditions very similar as for the basic model, there exists a unique BGP for a closed economy.

The analysis of open economies is very similar to that in basic model. In some of the formulas, for instance  $\mu$ , the counterpart of the parameter  $\alpha$  in the basic model is  $\alpha + \lambda$  in the extended model. See the details in Appendix B.

#### 5.2 Parameter Values

It is straightforward to discipline the value of some of the parameters using some basic and broad observations. Table 2 presents the resulting parameter values. I consider each period of life to represent 15 years so that fifteen years after starting working, a mid— or assistant manager promotes himself to the top position of a firm. Periods of 20 years probably capture better labor participations that range from ages in the mid-20s to ages in the mid-60s, but results do not change. The discount factor is set to  $\beta = (1.04)^{15}$  so that the annual interest rate is equal to 4%, roughly the historical average for the U.S. I set  $\omega = 0.1$  so that 10% of the population is in managerial occupations, 5% as top-managers and 5% as entrepreneurs. This values are in the in the low end of the numbers reported for related concepts in (e.g. entrepreneurship in Cagetti and De Nardi, 2006, managerial occupations in Eeckhout and Jovanovic 2010). Similarly, I set the

The weights  $\mu_z$  in  $Z^O$  are the shares of the country's aggregate supply of middle-managers used by firms with the different levels z of skills. These shares are different from the shares of labor services because, as shown below, the relative price paid for mid-managers and workers depends on the skills levels z of the top manager.

span-of-control  $\lambda + \alpha$  to 0.80, which is also in the low end of the values used in the literature (e.g. Buera and Shin 2010). Using low values for  $\omega$  and  $\lambda + \alpha$  leads to conservative estimates of the aggregate gains from openness. The individual values  $\lambda$  and  $\alpha$  are of no particular interest as long as (i) the occupation choice conditions  $\lambda/\varrho < \alpha < (1-\lambda)/2$  are satisfied, and (ii) the effect of  $\varrho^{\lambda}$  on the equilibrium G is undone with the calibration of the other parameters, which is precisely how I proceed.

	Parameter / Definition	Value	Criterion/target
$\omega$	fraction managers and entrepreneurs	0.1	% in managerial occupations
$\lambda$	output share, labor	0.725	see text
$\alpha$	output share, mid-managers	0.075	see text
$\beta$	discount factor	$(1.04)^{-15}$	annual risk free rate $\approx 4\%$
v	curvature, cost of skills	25	existence BGP all $\rho$ , $\gamma$
$v_0$	level of costs, skill acquisition	depends on $(\gamma, \rho)$	annual growth (BGP) $\approx 2\%$
$\gamma, \rho$	internalization, external diffusion	see text	comparative statics

**Table 2.** Parameter values for the quantitative exercises

As seen in Proposition 1, the existence of a BGP requires a high value for v, the curvature of the costs of skills  $\phi(\cdot)$ . I set v=25, a value high enough to insure the existence of a BGP for a wide range of the other parameters. While changing v affects the behavior of the economy, in the interest of space I shall focus on variations in  $\gamma$  and  $\rho$ , the internalization and external diffusion parameters. I experiment with different values of  $\gamma$  in [0,1] and values of  $\rho$  in  $(-\infty,\infty)$  to examine the implications of alternative specifications of the diffusion of knowledge.

Finally, given all the other parameter values, I re-calibrate the parameter  $v_0$  so that the equilibrium gross growth rate is  $G = (1.02)^{15}$ , i.e. a 2% implied net annual growth rate of output.

### 5.3 The Inflow of Foreign Know-how after Openness

Figure 4 shows the response of a closed economy previously in a BGP when it permanently opens up at t=1. For the interest of space, the illustrations consider only three parameter configurations: (a) a fully internalized diffusion case ( $\gamma=1$ ) where externalities are absent and the value of  $\rho$  is irrelevant; (b) a fully external case ( $\gamma=0$ ) with  $\rho=3.33$  to model a case in which externalities can have a strong impact on the accumulation of domestic know-how; and (c) an intermediate case ( $\gamma=0.75$ ,  $\rho=3.33$ ) where both externalities and internalized transfers are present. For all three cases, I assume that the initial domestic-to-foreign know-how ratio R=.7, which is high enough so that occupation choices do not bind upon openness.

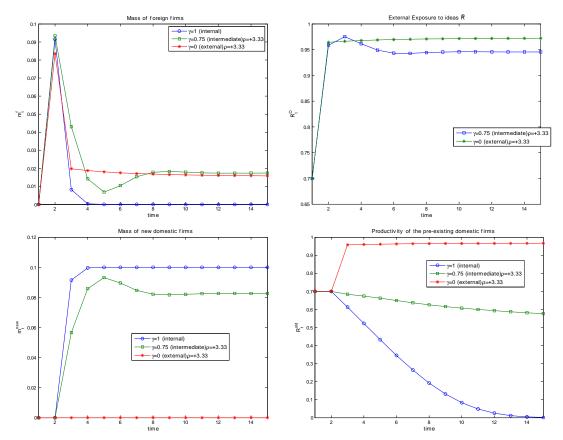


Figure 4: Foreign entry and diffusion of know-how after openness

In the three cases, as shown by the upper-left panel of figure 4, the initial lower cost of labor and mid-management services generates a burst of entry of foreign firms in the home country. This burst of entry is higher for parameter cases (a) and (c) because the better career opportunities offered by foreign firms increase even more the cost of middle-managers for domestic firms. When  $\gamma = 1$ , these learning opportunities are so strong that the new sector of domestic firms (lower-left panel) practically drive out foreign firms just after 3 periods (generations). On the opposite extreme, when  $\gamma = 0$  there is a sustained presence of foreign firms since  $R^O$  remains below 1 (upper-right panel) as the country does not fully catch up in terms of the exposure to ideas of the young generation of entrepreneurs.

The response in the intermediate case (c) captures aspects of both of the extreme cases (a) and (b). As in case (a), the entry of foreign firms generates a new sector of domestic firms (lower-left panel), but their know-how levels do not fully catch up with that of the foreign firms, as shown by see Figure 3 (for the limiting or BGP levels). Indeed, as shown by the lower-left panel of Figure 4, the size or total mass of these new domestic firms is lower than in the case (a). Because of this, as shown in the upper-left panel of that figure, foreign firms also have a sustained presence in the country; indeed, the mass of firms in the BGP is higher than in case (b) because when  $\gamma > 0$  foreign firms also offer better career prospects to mid-managers, which reduce their cost relative to what domestic firms must pay.

Perhaps the most interesting difference is in the impact of openness on the productivity of preexistent domestic firms. Not surprising, in the pure-externalities case (b), pre-existent domestic firms become more productive when their initial level is below the interior BGP ratio  $R^{\text{int}}$ . The opposite response, however, would be observed if initially  $R > R^{\text{int}}$ . Therefore, the pure externalities model appears completely at odds with the empirical results in Aitken and Harrison (1999), Xu (2000) and Alfaro et al. (2006) among others, which suggest that, if anything, the presence of foreign firms seem to have a negative impact in the productivity of domestic firms of developing countries.<sup>20</sup>

This evidence, in light of the results in the lower-right panel of Figure 4, supports the view that the diffusion of knowledge from foreign firms must involve, at least partially, some internalization, i.e.  $\gamma > 1$ . As shown by this figure, when  $\gamma = 1$  the response of pre-existing domestic firms is to significantly reduce their investments in know-how. This is consistent with the evidence cited above as openness leads to a rapid decline in the growth –and levels– of productivity for pre-existent firms. Notice that albeit less strong, this result is also valid when  $(\gamma = 0.75)$  as the negative impact of foreign competition more than compensate the positive effect of spillovers.

However, regardless of the specification of  $\gamma$  and  $\rho$ , a central message of the quantitative exercises is that openness always tends to push countries forward unless they are already closed to the leader. Once we set the span of control parameter  $\alpha + \lambda$  any where in the ballpark of the values used in the literature—and the value of v to insure the existence of a BGP in a closed economy— the interior BGP of an open economy is very close to full convergence (the only BGP when  $\gamma = 1$ ). Figure 2 in Section 3 had already shown this very clearly for the case of the pure externalities case  $\gamma = 0$ ; there, even if  $\rho = -10$ , the value of  $R^{\text{int}}$  was very close to 1.

Another simple message is that contrary to the implicit presumption in the empirical literature, externalities are neither sufficient nor necessary for openness to push the country forward. Here, it is important to make the distinction between the impact of openness on the pre-existing domestic firms and the impact on the country as a whole. Even if pre-existing domestic reduce their productivity—as the evidence suggests in some cases—, after openness the country may catches up as the pre-existent firms (or their progeny) are replaced by a new sector of more productive domestic firms. Indeed, more than pure externalities, some form of internalized transfers of knowledge seem to explain the emergence of new production sectors in Bangladesh, Colombia and Indonesia, as described by Rhee and Belot (1990).<sup>21</sup>

### 5.4 Welfare Gains from Openness

One of the central questions in this paper is whether openness enhances the aggregate welfare of a developing country. Using alternative configurations for the internalization and external diffusion of ideas, I compute the welfare gains for any initial domestic-to-foreign ratio of know-how R. I define

Welfare Gains of Openness 
$$\equiv \frac{\sum\limits_{t=0}^{\infty}\beta^tC_t^{\text{open}}}{\sum\limits_{t=0}^{\infty}\beta^tC_t^{\text{closed}}}-1,$$

<sup>&</sup>lt;sup>20</sup>As discussed by Xu (2000), Alfaro et. al. (2006) and Griffith et al (2002), the evidence suggest positive spillovers on domestic firms but only for developed countries. For developing countries some authors (e.g. Javorcik [2004] and Kugler [2005]) have argued for the existence of inter-industry spillovers, specifically, from foreign firms to local suppliers. However, productivity gains are probably better seen as internalized transfers, not spillovers, since as Javorcik herself reports, foreign firms in her sample were directly involved providing training, equipment and know-how to the local suppliers.

<sup>&</sup>lt;sup>21</sup>At the level of domestic industries, skill formation at the interior of the firm seems to be a major mechanism for aggregate skill formation and dissemination, as indicated by the empirical evidence that links the characteristics and the outcomes of parent firms with their spin-offs. For the U.S. car industry, Keppler (2001, 2002, 2006) documents that the genesis of the most successful car makers can be traced to former employees of other car makers. Agarwal et al (2004), Filson and Franco (2006) and Franco (2005) show the same for the rigid disk drive industry.

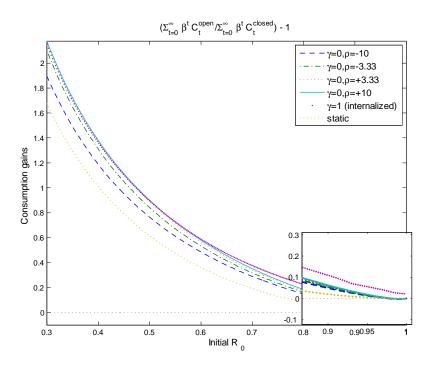


Figure 5: Welfare gains of openness under alternative initial conditions and parameters.

i.e. the net-gain in the present value of aggregate consumption attained by opening up at time t = 0. In every period, aggregate consumption is equal to the geographic output, minus foreign profits (retribution to foreign know-how) minus the investment costs of all young forming entrepreneurs working in foreign or domestic firms of all vintages. This definition of welfare gains, obviously, moves beyond the steady state calculations of Section 2, as it fully accounts for the transitional dynamics.

Figure 5 displays the welfare gains for initial values of R in the rage between .3 and 1. The figure reports the gains from the pure externalities models  $\gamma = 0$ , for values of the external diffusion parameter ranging from the very low  $\rho = -10$  to very high  $\rho = 10$ . It also reports the gains for the fully internalized formation  $\gamma = 1$ . As a benchmark, the figure also reports the 'static' gains, defined as the consumption gains that would accrue when skills are constant at the initial R, there are zero costs of investments, but foreign entrepreneurs fully appropriate their marginal contribution to the country's output.<sup>22</sup> The figure generates a number of important messages:

First, regardless of the parameter specification, the gains are very high, huge indeed for countries that lag far behind. For example, a country with only R = .3 of the productive knowledge as developed countries could attain gains above 2, i.e. they can consume more than 3 times being open than being closed; countries with R = .7 have gains in the order of 40% for all models. These numbers are large in light of the fact that gains fully account for the costs of building up the skills.

Second, as shown by the magnified region of the graph, even in the case of diffusion via externalities, the potential losses of openness are not only negligible but they can occur only in a very small interval around 1. Once we consider other potential benefits of openness for developed countries, the results of this paper strongly point in favor of openness for all countries.

Third, the gains of openness are much larger than the static gains. The positive impact on the

<sup>&</sup>lt;sup>22</sup>Static gains are coincide with the gains as computed by Burstein and Monge-Naranjo (2009) for the unilateral case assuming that all countries have the same country-embedded productivity.

formation of domestic know-how accounts for a large fraction of the gains, specially for the most backward countries. With openness, backward countries are exposed to the superior knowledge of develop countries; over time, this exposure to ideas help these countries to build up their own skills. Higher domestic skills eventually increase domestic consumption as less resources flow out of the country in the form of foreign profits. Notice that this is true even in the case when  $\gamma = 0$  and  $\rho = -10$ , a case in which the absorption of foreign ideas is hindered by the low level of domestic skills. Even in this case, for most values of R, learning adds about 20% of consumption relative to the static gains.

Four, even across the very different parameter specifications, the global behavior of the gains is very similar. This is remarkable, given the very different mechanisms by which models with pure externalities and models with fully internalized transfers of knowledge operate, as discussed in the previous section.

Finally, and perhaps most interestingly, the gains tend to be larger when diffusion of ideas is internalized than when it is via externalities. Only when R is low and  $\rho$  is as high, the gains are larger for the externalities model. Therefore, even if foreign firms only transfer knowledge to their own mid-managers and have no externalities on anyone else in the country, the country is better off being open. Moreover, notice than when  $\gamma = 1$ , the country always gains, no matter how close it is with the developed countries.

### 6 Concluding Remarks

The central questions in this paper were whether openness to foreign firms pushes a developing country to catch up with developed countries and whether developing countries would be better off pursuing this form of openness. To answer such questions, I constructed a model where the productivity of firms is driven by the entrepreneurial skills and in which these skills are built up every period on the basis of the ideas or knowledge implemented by domestic and foreign firms in the country. The model encompasses as special cases two standard –but conflicting– models for the diffusion of knowledge and provides a framework that clearly distinguishes the impact of foreign knowledge on pre-existing and new domestic firms. The simple quantitative exercises based on the model are surprisingly conclusive in that openness in fact pushes developing countries in the right direction and that, regardless of the specifics about the diffusion of knowledge, openness increase the welfare of countries.

A similar framework could be useful to study a number of other issues. First, the paper has focused on a rather drastic policy choice, either being completely closed or completely open. An interesting elaboration would be to characterize the optimal taxes or subsidies that a developing country would impose or grant foreign firms, considering their impact on the formation of domestic skills. The presence of externalities in the intertemporal formation of skills is likely to yield interesting trade-offs and time-consistency issues that may limit the ability of developing country to implement the optimal policy. Second, the analysis has considered entrepreneurial skills as the single engine of growth. The accumulation of complementary technology and other forms of human capital (e.g. Stokey 2010) and of physical capital could be important for the consequences of foreign entrepreneurial knowledge in developing countries. Third, the analysis is vertical in the sense that has focused on the impact of small developing countries. A horizontal two-country model, in which similar countries can gain from the knowledge of the other, either by engaging in trade and multinational activity, seems to be the way to go to understand the gains of openness for developed countries.

The emphasis of this paper has been on the aggregate consequences of openness to international firms. An interesting avenue for future research is to use a model similar to one developed here to extend the analysis of the efficiency and welfare consequences of financial frictions (as in Buera and Shin 2010) and tax policies (as in Cagetti and De Nardi 2006) that distort the endogenous formation of entrepreneurial know-how in a closed economy.

### A Analytical Aspects of the Basic Model

**Proof of Lemma 1** The value of V may be  $+\infty$  as illustrated in the proof for Proposition 1 for the case of homogeneous firms. In such a case, the function  $w(\cdot)$  is not well defined. However, if V is bounded, then the monotonicity of the function  $w(\cdot)$  exists arises directly from the envelope condition on V. Next, the monotonicity of  $z'(\cdot)$  arises from the fact that it is given by

$$\beta \left(\alpha z'\right)^{\frac{\alpha}{1-\alpha}} g\left(z'\right) = v_0 \left(\frac{z'}{z^E}\right)^v,$$

where  $g\left(z'\right) \equiv w'\left(z'\right)^{\frac{-\alpha}{1-\alpha}} \left[1 + \frac{\alpha\gamma v v_0 \left(z''/\left(z^E\right)'\right)^{1+v} \left(z'\right)^{\gamma} \left(z^{O'}\right)^{1-\gamma}}{(1+v)w(z')}\right]$ , which is non-decreasing in z'. For a maximization, the

left-hand-side must cross the right-hand side from above, hence the need for the condition  $v > \alpha/(1-\alpha)$ . Thus, a z or higher  $z^E$  leads to a higher optimal value for z'. Finally, straightforward differentiation leads to

$$\frac{\partial \ln{(z')}}{\partial \ln{(z^E)}} = \frac{v}{\left[v - \frac{\alpha}{1-\alpha}\right] - \frac{\partial g(z')}{\partial z'} \frac{z'}{g(z')}} \ge \frac{v}{v - \frac{\alpha}{1-\alpha}} > 1,$$

where the first inequality holds because  $\frac{\partial g(z')}{\partial z'} \frac{z'}{g(z')} \ge 0$  and the second because  $v > \alpha/(1-\alpha)$ . Since  $z^E = z^{\gamma} \left(Z^O\right)^{1-\gamma}$ , then,

$$\frac{\partial \ln \left(z'\right)}{\partial \ln \left(z^{E}\right)} \geq \frac{\gamma v}{v - \frac{\alpha}{1 - \alpha}},$$

which is greater than one if  $\gamma > 1 - \frac{\alpha}{(1-\alpha)v}$ , as stated.

Proof of Proposition 1. Define  $L(G) \equiv \beta \left[1 + \frac{vv_0\gamma}{1+v} (G)^{1+v}\right]$  the marginal return to investing (the right- and left-hand sides of the equation 10) and  $R(G) \equiv v_0(G)^v$  the marginal cost of investing (the right-hand-side). Notice that L(0) > 0 = R(0) and that the curvatures of  $L(\cdot)$  and  $R(\cdot)$  are 1+v and v, respectively. Therefore,  $L(\cdot)$  may lay above  $R(\cdot)$  for all positive real numbers. In such a case, the optimal investment would be degenerate,  $G = +\infty$ , and the career value V for a young person would be unbounded,  $V = +\infty$ . However,  $L(\cdot)$  may lay below  $R(\cdot)$  for some range; if so, because of its higher curvature, it would cross  $R(\cdot)$  twice, first from above and then from below. The second root, however, is not relevant because it is a local minimum. Therefore, (10) has at most one valid fixed point, which proves (b). To prove (a) first, if  $\gamma = 0$ , then  $G = (\beta/v_0)^{1/v}$  is the unique solution; the inequality  $\beta < [v_0(1+v)]^{\frac{1}{1+v}}$  insures that  $\phi(G) < 1$  and therefore, the growth in skills is not so high so as to lead to negative aggregate consumption. For  $\gamma > 0$ , from the intermediate value theorem, a sufficient condition for  $L(\cdot)$  to cross  $R(\cdot)$  is to find a point G > 0 for which L(G) < R(G). It is straightforward to show that the two curves are parallel to each other, i.e.  $\partial L/\partial G = \partial R/\partial G$ , only at the point  $\bar{G} = (\beta\gamma)^{-1}$ , and obviously,  $\partial L/\partial G < (>) \partial R/\partial G$  for  $G < (>) \bar{G}$ . It is also straightforward to show that  $L(\bar{G}) < R(\bar{G})$  if  $\beta < (v_0/[\gamma^v(1+v)]]^{\frac{1}{1+v}}$ . The inequality  $[v_0/(1+v)]^{\frac{1}{1+v}}/\gamma < \beta$  is equivalent to  $\phi(\bar{G}) < 1$ , which insures that the first crossing G (the BGP) does not imply negative aggregate consumption. (c) The inequality  $\beta > v_0(1+v)/(1+v+v_0\gamma)$  is equivalent to L(1) > R(1) which insures that the first crossing, if it exists, is higher than 1. (d) Solving equation (10) for  $G_{t+1}$ 

$$G_{t+1} = \left[ \frac{1+v}{\gamma v v_0} \left( \frac{v_0}{\beta} \left( G_t \right)^v - 1 \right) \right]^{\frac{1}{1+v}},$$

which is a strictly concave function of  $G_t$  that first crosses the 45° line from below, precisely at the valid BGP G. The implied  $\{G_{t+1}\}$  from any  $0 < |G_0 - G^{BGP}| < \epsilon$  would diverge. Therefore, the only possibility is  $G_t = G^{BGP}$  for all t. (e) If  $\gamma = 0$ , then  $z^E = Z^O$  for all, regardless of the initial distribution. In this case, the economy converges to a uniform entrepreneurs BGP in which  $z_{t+1} = (\beta/v_0)^{1/v} Z_t^O$ . I now consider  $\gamma > 1 - \alpha/\left[(1-\alpha)v\right]$ , consider any  $z_0 < z_1$  and denote by  $z_i^t$  the skills levels of the t generation of a dynasty of entrepreneurs starting with  $z_i$  for i = 0, 1. From Lemma 1,  $\lim_{t \to \infty} z_0^t/z_0^t = 0$ . Given an initial mass  $M_i^0$  of entrepreneurs of each type i, the mass  $M_i^t$  at time t equals  $M_0^i \prod_{t=0}^{t-1} n_i^t$ , where  $n_i^t$  is the labor employed at time  $\tau$  by the dynasty i. Since  $w_\tau$  (·) is non-increasing then  $n_0^\tau/n_1^\tau \le (z_0^\tau/z_1^\tau)^{\frac{1-\alpha}{1-\alpha}}$ , which is a

decreasing sequence that converges to 0. With it  $\lim_{t\to\infty} M_0^t/M_1^t = 0$ . Since this holds for any pair  $z_0 < z_1$ , the only possibility is that the dynasty starting with the highest initial skill level takes over the entire population.

**Proof of Proposition 2.** Part (a) is straightforward: since R=1, the equilibrium conditions for the closed economy BGP apply to the open economy with  $m_f=0$  because  $w_h=w_f$ . Part (b) is also straightforward. Assume that there is a BGP with  $R_j \leq 1$  with the inequality strict for some  $j < \infty$ . Since the economy is open,  $w_h=w_f$ , and  $m_f > 0$ , otherwise, there would be excess supply of labor because  $n_j \leq 1$  with strict inequality for the j s for which  $R_j < 1$ . Now, if  $\gamma = 1$ , and by definition  $R_0 = 1$ , then  $R_0^E = 1$  and in equilibrium  $R_1 = 1$ . Similarly, if  $\gamma < 1$  but  $\rho = +\infty$ , then  $R_0^O = 1$  implying that  $R_0^E = 1$ , and then  $R_1 = 1$ . In either case, whenever  $R_j = 1$ , it is the case that  $R_j^E = 1$  and  $R_{j+1} = 1$ . Therefore, by induction  $R_j = 1$  for all  $j = 1, 2, ... \infty$  and the result is established. For part (c) recall that the transition function is

$$R_{\text{next}} = F(R) \equiv \left[ 1 + (R)^{\rho + \frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}} \right]^{\frac{\mu}{\rho}}$$

a twice continuously differentiable function. It is easy to verify that F(1) = 1 and that

$$F(0) = \begin{cases} 1 & \text{if } \rho > -1/(1-\alpha), \\ 2^{\frac{-\mu}{1-\alpha}} \in (0,1) & \text{if } \rho = -1/(1-\alpha), \\ 0 & \text{if } \rho < -1/(1-\alpha). \end{cases}$$

Moreover, the first derivative is

$$F'\left(R\right) = \left(\frac{\mu}{\rho}\right) \left[1 + (R)^{\rho + \frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho} - 1} \times \left[\left(\rho + \frac{1}{1-\alpha}\right) R^{\rho + \frac{\alpha}{1-\alpha}} - \left(\frac{1}{1-\alpha}\right) R^{\frac{\alpha}{1-\alpha}}\right],$$

which is obviously a continuous function, and moreover, regardless of the parameter values,  $F'(1) = \mu > 1$ . Therefore, there is an  $\varepsilon_1 > 0$  (which may depend on the parameter values) such that F(R) < R for all  $R \in (1 - \varepsilon_1, 1)$ . Therefore, whenever  $\rho \ge -1/(1-\alpha)$  the intermediate value theorem implies the existence of an interior BGP because there is an  $R \in (0,1)$  such that F(R) = R. For the case of  $-\infty < \rho < -1/(1-\alpha)$ , using de l'Hôpital's rule, it is easy to verify that  $\lim_{R\to 0} F'(R) = +\infty$ . Then, there exist an  $\varepsilon_0 > 0$  (which may depend on the parameter values) such that F(R) > R for all  $R \in (0,\varepsilon_0)$ . Then, again from the intermediate value theorem the existence of the interior BGP is established for these set of parameters.

Now, define 
$$H(R) \equiv 1 + (R)^{\rho + \frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}$$
. Obviously  $F(R) = H(R)^{\frac{\mu}{\rho}}$ ,  $F'(R) = \left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-1\right)} H'(R)$  and

$$F^{\prime\prime}\left(R\right)=\left(\frac{\mu}{\rho}\right)H\left(R\right)^{\left(\frac{\mu}{\rho}-2\right)}\left\{H\left(R\right)H^{\prime\prime}\left(R\right)+\left(\frac{\mu}{\rho}-1\right)\left[H^{\prime}\left(R\right)\right]^{2}\right\},$$

where I have taken the term  $\binom{\mu}{\rho}H(R)^{\left(\frac{\mu}{\rho}-2\right)}$  as common factor and then simplified. This common term is positive for all R>0 hence  $sign\left\{F''(R)\right\}=sign\left\{H(R)H''(R)+\left(\frac{\mu}{\rho}-1\right)\left[H'(R)\right]^2\right\}$ . Since  $\left[H'(R)\right]^2$  is obviously positive, the term  $\left(\frac{\mu}{\rho}-1\right)\left[H'(R)\right]^2$  is always positive, zero or negative, depending on whether  $\frac{\mu}{\rho} \gtrsim 1$ . The function H(R) is always positive on [0,1]. Then, F''(R) can only change signs if  $H''(\cdot)$  changes signs. The unique inflection point of  $H(\cdot)$ , where it changes from concave to convex or vice-versa is  $R^I=\left\{\alpha/\left[(1-\alpha)\left(\rho^2\left(1-\alpha\right)+\alpha\rho\right)\right]\right\}^{\frac{1}{\rho}}$ , which may or may not fall in the interval [0,1]. In any event, the transition function  $F(\cdot)$  has at most one inflection point, which is (larger than) [smaller than]  $R^I$  if  $\frac{\mu}{\rho}=1$  ( $\frac{\mu}{\rho}<1$ )  $[\frac{\mu}{\rho}>1]$ . Because  $\mu>1$ , if  $F(\cdot)$  changes curvature it is necessarily from concave to convex and since  $\lim_{R\to 1} F'(R)=\mu>1$ , then,  $F(\cdot)$  crosses at most once the  $45^0$  line in the interval [0,1].

Finally, to establish global stability of the interior BGP, recall that existence was established by showing F(R) > R for some a lower sub-interval of [0,1] and F(R) < R for a higher sub-interval. Because of this, the continuity of F and the fact that its crossing is unique, then  $F(\cdot)$  must cross the  $45^{\circ}$  from above, which implies the global stability of the BGP.

**Proof of Lemma 2.** If  $\beta(1-2\alpha) \geq v_0 G^v/(1+v)$ , then when all the initial old have the same skill level Z: (a) all of the old are better off being active entrepreneurs and (b) all young individuals would be better off investing and become entrepreneurs in the next period. Therefore, optimal occupation choices are satisfied, and, by assuming all the parameter conditions of Proposition 1, then the allocations are an equilibrium. Alternatively, if  $\beta(1-2\alpha) < v_0 G^v/(1+v)$ , if all the young invest, then the net present value  $w - Z\phi(G) + \beta G\pi$  would be strictly below the option  $w + \beta Gw$  of not investing and remaining a worker next period. If only  $\omega$  of the young invest, then, in the BGP, the aggregate supply of labor will be  $2-\omega$  (all the mass 1 of young individuals plus the mass  $1-\omega$  of the old who did not invest) and the supply of entrepreneurs will be  $\omega$ . With homogeneous entrepreneurs with skills Z, workers' wages are equal to  $w = \alpha \left[ (2-\omega)/\omega \right]^{\alpha-1} Z$  and entrepreneurs' earnings are  $\pi = (1-\alpha)\left[ (2-\omega)/\omega \right]^{\alpha} Z$ . By construction, the condition

$$\beta\left(\frac{2-\omega}{\omega}\right)^{\alpha}\left(\frac{2\left(1-\alpha\right)-\omega}{2-\omega}\right) = \frac{v_0}{1+v}G^v,$$

implies that the fraction  $\omega$  makes the a young individual to be indifferent between remaining a worker or investing at the rate G and becoming an entrepreneur; likewise, the first root of

$$\beta \left(\frac{2-\omega}{\omega}\right)^{\alpha} \left(1 + \frac{\gamma v v_0}{(1+v)} G^{1+v}\right) = v_0 G^v.$$

implies that for those investing, the growth rate G is optimal, given than only  $\omega$  of the current young are investing.

### B The Model with Middle-Managers and Workers

Given a wage for the workers w, and a wage function  $w_m(\cdot)$  for middle-managers, the earnings of an entrepreneur with skills z are

$$\pi\left[z;w_{m}\left(z\right),w\right]\equiv\max_{\left\{ n,l\right\} }\left\{ zn^{\alpha}l^{\lambda}-w_{m}n-wl\right\} =\left(1-\alpha-\lambda\right)z^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{w_{m}\left(z,Z\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\left(\frac{\lambda}{w}\right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Consider first a closed economy in which all entrepreneurs have skills  $Z_h = Z > 0$ . From the market-clearing conditions, n = 1 and  $l = \varrho$ , the wage for workers is  $w = \lambda \varrho^{\lambda - 1} Z$  and an the wage function  $w_m(z, Z)$  must satisfy  $w_m(Z, Z) = \alpha Z \varrho^{\lambda}$ . In this case, the earnings of entrepreneurs are  $\pi = \pi(Z, Z) = (1 - \alpha - \lambda) Z \varrho^{\lambda}$ . for any other skills level z, given Z,  $Z_{t+1}$   $w_{m,t+1}(z', Z_{t+1})$ , and  $w_{t+1}$ , the entrepreneur would have to pay his mid-managers wages equal to

$$w_{m}(z,Z) = w_{m}(Z,Z) + V[Z, w_{m,t+1}(\cdot), w_{t+1}] - V[z^{\gamma}Z^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1}]$$
(26)

which compensate for better or worse learning possibilities. Here, similar to the basic model, the learning or career value  $V(\cdot)$  of a mid-management job with exposure  $z^E$  of ideas is

$$V\left[z^{E}, w_{m,t+1}\left(\cdot\right), w_{t+1}\right] \equiv \max_{z'} \left\{\beta\pi\left[z', w_{m,t+1}\left(z', Z_{t+1}\right), w_{t+1}\right] - z^{E}\phi\left(z'/z^{E}\right)\right\}. \tag{27}$$

If it is well defined, which requires that  $v > (\alpha + \lambda) / (1 - \alpha - \lambda)$ , the optimal skill accumulation z' for a mid-manager who will go on to be an entrepreneur is given the FOC

$$\beta \left[ \pi_1 \left[ z', w_{m,t+1} \left( z', Z_{t+1} \right), w_{t+1} \right] + \pi_2 \left[ z', w_{m,t+1} \left( z', Z_{t+1} \right), w_{t+1} \right] \frac{\partial w_{m,t+1} \left( z', Z_{t+1} \right)}{\partial z'} \right] = \phi' \left( \frac{z'}{z^E} \right), \tag{28}$$

where  $\pi_1$  (·) and  $\pi_2$  (·) denote, respectively, the derivative of  $\pi$  with respect to the manager's skill z and with respect to the wage  $w_{m,t+1}$  (z, Z) that he must pay his mid-managers. These derivatives are given by

$$\pi_{1}\left(\cdot\right)=z^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}\left(\frac{\alpha}{w_{m}\left(z,Z\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\left(\frac{\lambda}{w}\right)^{\frac{\lambda}{1-\alpha-\lambda}}\text{ and }\pi_{2}\left(\cdot\right)=-z^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{w_{m}\left(z,Z\right)}\right)^{\frac{1--\lambda}{1-\alpha-\lambda}}\left(\frac{\lambda}{w}\right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Finally, from (26),  $\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}\left(z''\right)\right] \frac{\partial \left(z^E\right)'}{\partial z'}$ , and applying the envelope condition on V one period ahead:

$$V_{1}\left[\left(z^{E}\right)^{t+1},w_{m,t+2}\left(\cdot\right),w_{t+2}\right]=-\frac{vv_{0}}{1+v}\left(\frac{z^{t+2}}{\left(z^{E}\right)^{t+1}}\right)^{1+v},$$

where I use simplifications derived from the functional form assumed for  $\phi(\cdot)$ .

**BGP closed economies.** Impose  $z_t = z_t^E = Z_t$ , and  $G = Z_{t+1}/Z_t$  for all periods t. Plugging  $w_{t+1} = \lambda \varrho^{\lambda-1} Z_{t+1}$  and  $w_{m,t+1}(Z,Z) = \alpha \varrho^{\lambda} Z_{t+1}$ , we get, that the derivatives boil down to  $\pi_1 = \varrho^{\lambda}$  and  $\pi_2 = -1$ ; we also get  $V_1 = -\frac{vv_0}{1+v}(G)^{1+v}$ ,  $\frac{\vartheta(z^E)'}{\vartheta z'} = \gamma$ , and that  $\phi'\left(\frac{z'}{z^E}\right) = v_0 G^v$ . Using all of these in (28), the resulting expression is

$$\beta \left( \varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v} \right) = v_0 G^v,$$

which is exactly the condition in the text.

The reminder of equilibrium conditions (ex-ante and ex-post optimality of occupation choices) follow exactly the same lines as discussed above for the basic model (i.e.  $\lambda = 0$  and  $\omega = 1$ ).

The definition of  $\mu$  for this model is  $\mu = v/\left[v - (\alpha + \lambda)/(1 - \alpha - \lambda)\right]$  which is greater than one when  $v > (\alpha + \lambda)/(1 - \alpha - \lambda)$  which is assumed throughout.

**BGP open economies.** Home being small and open free entry and foreign being in a BGP, implies that at home the wage of workers is equal to  $w = \lambda \varrho^{\lambda-1} Z_f$ , and that foreign firms pay domestic mid-managers wages equal to  $w_{m, t+1}(Z_f) = \alpha \varrho^{\lambda} Z_f$ . Then, a domestic firm with skills z must pay a wage

$$w_{m}(z, Z_{f}) = w_{m}(Z_{f}) + V\left[ (Z_{f})^{\gamma} \left( Z^{O} \right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1} \right] - V\left[ z^{\gamma} \left( Z^{O} \right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1} \right],$$
 (29)

where  $Z^O$  is the outside set of ideas circulating in the country, which must be determined as part of the equilibrium. Let  $R = z/Z_f$  and denote by d(R) the compensating differential, relative to  $Z_f$ , of an entrepreneur with skills  $z = RZ_f$ . That is,  $w_m(z, Z_f) = \left[\alpha \varrho^{\lambda} + d(z/Z_f)\right] Z_f$ , where to shorten the notation I have omitted indicating the dependence on  $Z^O$ . After simplifying, the derivatives  $\pi_1$  and  $\pi_2$  become

$$\pi_{1} = \Phi R^{\frac{\alpha + \lambda}{1 - \alpha - \lambda}} \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{\alpha}{1 - \alpha - \lambda}} \text{ and } \pi_{2} = -\Phi R^{\frac{1}{1 - \alpha - \lambda}} \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{1 - \lambda}{1 - \alpha - \lambda}}.$$
 (30)

where  $\Phi \equiv [\varrho]^{\frac{\lambda(1-\lambda)}{1-\alpha-\lambda}}$ . Next, using the fact that  $Z_{f,\ t+2}/Z_{f,\ t+1} = G$ , defining  $R^E \equiv z^{\gamma} \left(Z^O\right)^{1-\gamma}/Z_f = R^{\gamma} \left(R^O\right)^{1-\gamma}$  –as I defined in the text for the basic model– then

$$V_1 \left[ \left( z^E \right)^{t+1}, w_{m,t+2} \left( \cdot \right), w_{t+2} \right] = -\frac{v v_0}{1+v} \left( G \frac{R_{t+2}}{R_{t+1}^E} \right)^{1+v}, \tag{31}$$

where  $R_{t+1}^E$  is the relative exposure to ideas that the entrepreneur will provide to his mid-managers in the next period and  $R_{t+2}$  is their relative accumulation of skills. It is also straightforward to compute,

$$\frac{\partial \left(z^{E}\right)^{t+1}}{\partial z^{t+1}} = \gamma \frac{R_{t+1}^{E}}{R_{t+1}} = \gamma \left(\frac{R^{O,t+1}}{R^{t+1}}\right)^{1-\gamma}.$$
(32)

Finally,

$$\phi'\left(\frac{z_{t+1}}{z_t^E}\right) = v_0 \left[G\frac{R_{t+1}}{R_t^E}\right]^v. \tag{33}$$

Plugging conditions (30), (31), (32) and (33) into condition (28), implies that the optimal accumulation of skills  $R_{t+1}$  for a worker of a mid-manager of an entrepreneur with relative skills  $R_t$ 

$$\beta\Phi R_{t+1}^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}\left[\left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}+\frac{\gamma v v_0 R_{t+1}^E}{1+v}\left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R_{t+1}\right)}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}\left(G\frac{R_{t+2}}{R_{t+1}^E}\right)^{1+v}\right]=v_0\left[G\frac{R_{t+1}}{R_t^E}\right]^v,$$

where I have grouped and simplified terms involving  $R_{t+1}$ . Because  $R_{t+1}^E$  obviously depends on  $R_{t+1}$ , it is instructive to re-write this condition as

$$\beta \Phi R_{t+1}^{\frac{\alpha+\lambda}{1-\alpha-\lambda}} \left[ \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d\left(R_{t+1}\right)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} + \frac{\gamma v v_0}{1+v} \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d\left(R_{t+1}\right)} \right)^{\frac{1-\lambda}{1-\alpha-\lambda}} \frac{\left(G R_{t+2}\right)^{1+v}}{\left(R_{t+1}\right)^{\gamma v} \left(R_{t+1}^{O}\right)^{(1-\gamma)v}} \right] = v_0 \left[ G \frac{R_{t+1}}{R_t^E} \right]^{v}. \quad (34)$$

Notice that re-interpreting the index t not as a period but as a 'vintage' instead, this condition determines the time invariant  $\{R_j\}_{j=1}^{\infty}$  for an open economy BGP. Also, notice that forcing  $\lambda = 0$ , the condition boils down to the equation in the text for basic model.

Now, to compute the compensating differentials d(R), first observe that the earnings of an entrepreneur with skills z can be written as

$$\pi(z, Z_t) = Z_f \left[ \Phi(1 - \alpha - \lambda) R^{\frac{1}{1 - \alpha - \lambda}} \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{\alpha}{1 - \alpha - \lambda}} \right].$$

Let let  $R_{t+1}^1$  denote the relative skills of domestic entrepreneur who was trained at time t in a foreign firm. Then, the career value for a mid-manager in a foreign firm is

$$V_{f,t} = Z_{f,t} \left[ \beta G \Phi \left( 1 - \alpha - \lambda \right) \left( R_{t+1}^1 \right)^{\frac{1}{1 - \alpha - \lambda}} \left( \frac{\alpha}{\alpha \varrho^{\lambda} + d \left( R_{t+1}^1 \right)} \right)^{\frac{\alpha}{1 - \alpha - \lambda}} - \left( R_t^O \right)^{1 - \gamma} \frac{v_0}{1 + v} \left( \frac{G R_{t+1}^1}{\left( R_t^O \right)^{1 - \gamma}} \right)^{1 + v} \right],$$

and his life-time utility is  $\alpha \varrho^{\lambda} Z_{t+1} + V_{f,t}$ . On the other hand, working for a domestic firm with relative skills R, the mid-manager would attain

$$Z_{f,t}\left[\alpha\varrho^{\lambda}+d\left(R\right)+\beta G\Phi\left(1-\alpha-\lambda\right)\left(R_{t+1}^{R}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R_{t+1}^{R}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}-\frac{v_{0}}{1+v}\frac{\left(GR_{t+1}^{R}\right)^{1+v}}{\left(R^{\gamma}\left(R_{t}^{O}\right)^{1-\gamma}\right)^{v}}\right]$$

where  $R_{t+1}^R$  indicates the optimal skill accumulation –that solves (34)– when exposed to any level R. Equating this expression with the life-time utility attainable working for a foreign firm, then

$$\begin{split} d\left(R\right) &= \beta G\Phi\left(1-\alpha-\lambda\right) \left[ \left(R_{t+1}^{1}\right)^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R_{t+1}^{1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \left(R_{t+1}^{R}\right)^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R_{t+1}^{R}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} \right] \\ &+ \left(\frac{v_{0}}{1+v}\right) \frac{G^{1+v}}{\left(R_{t}^{O}\right)^{(1-\gamma)v}} \left[ \frac{\left(R_{t+1}^{R}\right)^{1+v}}{R^{\gamma v}} - \left(R_{t+1}^{1}\right)^{1+v} \right]. \end{split}$$

Instead of solving for any arbitrary levels R, we can restrict this equation for the time-invariant vintage structure in a BGP for an open economy using j to index each vintage, i.e.  $d_j \equiv d(R_j)$ . Notice that this is a generalization of the expression in the text. First, it does not presume  $\lambda = 0$ ; if it did, we recover the expression in the text for the basic model. Second, it allows the computation for time varying  $\{R_j^t, d_j^t, R^{O,t} : j \geq 1, t \geq 0\}$ . Indeed, on the basis of these equations, I use a simple shooting algorithm on  $R^{O,t}$  to compute for the transition dynamics of a closed economy starting in a BGP to the final steady state after openness.

Finally, the optimal amount of labor and mid-management units hired by a domestic entrepreneur with  $0 \le R \le 1$ , are equal to

$$l\left(R\right) = R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha\varrho^{\frac{(1-\alpha)(1-\lambda)}{\alpha}}}{\alpha\varrho^{\lambda} + d\left(R\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} \text{ and } n\left(R\right) = R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha\varrho^{\lambda}}{\alpha\varrho^{\lambda} + d\left(R\right)}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}.$$

Notice that the ratio n(R)/l(R) is increasing in R when d(R) is decreasing. The relative cost of mid-management services is lower for more productive as they offer better learning opportunities.

In a BGP of an open economy, the amount of mid-managers hire by entrepreneurs of vintage j (foreign firms are j=0)

$$n_j = (R_j)^{\frac{1}{1-\alpha-\lambda}} \left( \frac{\alpha \varrho^{\lambda}}{\alpha \varrho^{\lambda} + d_j} \right)^{\frac{1-\lambda}{1-\alpha-\lambda}},$$

and the mass of domestic mid-managers in each vintage j are

$$\hat{m}_j = \hat{m}_0 \prod_{i=0}^{j-1} n_i.$$

To clear the domestic market for mid-managers, i.e.  $\sum_{j=0}^{\infty} \hat{m}_j n_j = \omega$ , the total mass entry of foreign firms must be  $\hat{m}_0 = \omega / \left[ \sum_{k=0}^{\infty} \prod_{j=0}^{k} n_j \right]$ . Then, the shares (weights in  $R^O$ ) are  $m_j = \hat{m}_j / \omega$ .

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