

# The Inefficiency of Refinancing: Why Prepayment Penalties Are Good for Risky Borrowers\*

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## Abstract

This paper explores the practice of mortgage refinancing in a dynamic competitive lending model with risky borrowers and costly default. We show that prepayment penalties are often welfare improving, and that they are more beneficial to borrowers with higher risk of default. Mobility mitigates the benefits of prepayment penalties only for the safest borrowers. Empirical evidence from more than 43,000 securitized fixed rate mortgages (FRMs) supports the predictions of the model. First, borrowers who receive positive credit shocks are much more likely to prepay their mortgage, the riskier they are. Second, we show that less creditworthy borrowers are the most likely to have prepayment penalties, but also receive appreciable benefits. Subprime borrowers with FICO scores below 620 obtain rates as much as 0.7% lower than similar borrowers with fully prepayable mortgages and default at a lower rate (13% default rate) than comparable borrowers with no prepayment penalties (18% default rate). Our findings suggest that regulations banning refinancing penalties might have the unintended consequence of raising interest rates, increasing mortgage default, and limiting available credit for the riskiest borrowers.

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# 1 Introduction

Prepayment penalties are widely used in the mortgage market and are often highly concentrated among the least creditworthy borrowers<sup>1</sup>. Recently, they have generated great controversy and criticism resulting from an increase in delinquencies and defaults among residential borrowers. Critics contend that prepayment penalties can hurt borrowers already saddled with high interest payments and accuse mortgage originators of predatory lending to naive borrowers who do not fully understand mortgage terms. In response to these concerns, US House Representatives Watt, Miller, and Frank introduced the Mortgage Reform and Anti-Predatory Lending Act of 2007, which proposes limits on prepayment penalties, especially for high cost mortgages. New rules proposed by the Federal Reserve Board on December 2007 for public comment also restrict the use of prepayment penalties. Despite the criticism and threats of regulation, mortgage originators often defend prepayment penalties as a way to ensure that they can make mortgages available for high risk borrowers.

In this paper, we explore the practice of mortgage refinancing, considering why borrowers might choose to prepay a mortgage in response to a credit shock. We begin by developing a simple dynamic competitive lending model with fixed rate mortgage contracts (FRMs). Within the model, we consider borrowers who are ex ante identical other than differences in initial wealth (a measure of credit quality). Borrowers may choose to obtain a FRM to purchase a home. Homeownership is assumed to generate positive utility gains for the borrowers. Once a mortgage is originated, the borrower's creditworthiness evolves stochastically over time. Borrowers who are hit by a series of severe enough financial shocks will default.<sup>2</sup> (Default is assumed to be costly for both borrowers and lenders.)

Our model allows us to solve for the equilibrium interest rate in regimes with and without prepayment penalties assuming no change in interest rates. The mortgage premium for a FRM increases as observed credit quality falls. Higher mortgage premiums compensate the lender for larger expected losses due to higher defaults by borrowers with the worst credit.

In the model with FRM's and constant mortgage rates, borrowers choose to refinance only to obtain a mortgage with a lower risk premium. Over time, the borrowers who choose to refinance are those whose creditworthiness has improved. Thus mortgage pools are increasingly composed of the least creditworthy borrowers; that is, borrowers who have received flat or negative credit shocks since mortgage origination. A rational lender anticipates this adverse selection over time and will compensate for it by charging a higher premium on the loans that are freely prepayable.

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<sup>1</sup>According to a Standard & Poor's survey, as many as 80% of mortgages in the subprime market carried prepayment penalties in mid-2000, up from 50% in 1997. In our sample of securitized fixed rate mortgages, we find that the incidence of prepayment penalties ranges from about 60% of subprime mortgages to about 2.5% of prime mortgages.

<sup>2</sup>This result is consistent with empirical evidence pointing that consumer delinquency problems are mainly the result of unexpected negative events, that neither the lender nor the borrower could have anticipated at the time the credit request was evaluated (see for example Getter (2003)). In the words of Amy Crews Cutts, deputy chief economist for Freddie Mac, cited by NYT (2007): "If you come in at the edge of affordability, and gas prices go up \$100 a month, and insurance premiums go up, and then a water heater breaks, that's the kind of thing that can put a family over the edge."

In our model, a prepayment penalty acts as a commitment device that allows the borrower to credibly remain with the same lender for a longer period of time. We show that this commitment is valuable as it allows the lender to use borrowers with good ex-post credit shocks to cross subsidize borrowers with poor ex-post credit shocks. Refinancing, on the other hand, generates two types of inefficiencies in mortgage lending market, both of them due to the fact that lenders must charge ex-ante higher mortgage rates for fully prepayable mortgages. First, the higher ex-ante mortgage premium makes the ex-post less creditworthy borrowers (those who received negative financial shocks) more likely to default, which is socially costly. Second, the required increase in premium to compensate for refinancing leads some particularly high-risk borrowers to be excluded from the credit markets, although these borrowers would otherwise be able to qualify for a loan if refinancing were not allowed (e.g., if prepayment penalties were employed). Both of these effects reduce welfare. Therefore, we conclude that prepayment penalties benefit borrowers, with the riskiest borrowers benefitting the most.

Our result on the inefficiency of refinancing is similar in spirit to the observation that lack of consumer commitment can generate inefficiencies in life or health insurance markets as the short-term contracts do not offer insurance against "reclassification risk"<sup>3</sup>: bad news about the persistent health status of a consumer can result in increased premiums. From an ex-ante perspective, commitment to a long term-contract can provide insurance against reclassification risk and thus be welfare improving.

We examine the predictions of our model using data from securitized mortgages based on data from Loan Performance. In order to match the model as closely as possible and to limit the likelihood of refinancing driven by lower interest rates, we generate a sample of more than 43,000 fixed rate mortgages originated in a single month (June, 2003) when interest rates were at their lowest in the last thirty years. We also focus exclusively on FRMs so as to avoid empirical complications in those cases where borrowers choose to refinance in order to avoid a step-up in mortgage rates when an initial teaser rate expires or when short-term interest rates rise.

The empirical work supports key predictions of our model. First, using house price shocks as a proxy for ex-post wealth shocks, we find that borrowers who receive positive house price shocks are much more likely to prepay their mortgage than borrowers in locations where house prices grew less quickly. When we decompose house price appreciation into an explained and residual component (based on a model that uses lagged house price growth to explain future house price changes), we find that the shocks to house price appreciation are relatively more important in explaining prepayments. Moreover, the prepayment rate of the highest risk borrowers is the most sensitive to house price changes, whether we examine overall house price appreciation or just shocks to the appreciation rate. In response to a one standard deviation increase in house price appreciation, subprime borrowers are about 20 percent more likely to prepay their mortgages. By contrast, relatively safe prime borrowers increase their likelihood of paying off their mortgage by only

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<sup>3</sup>Cochrane (1995) refers to "reclassification risk" as "premium risk". See for example Hendel and Lizzeri (2003) for a recent treatment in the context of life insurance.

about 5 percent in response to the same one standard deviation increase in house price appreciation.

Next we limit our sample to subprime borrowers and examine the use of prepayment penalties as well as the impact of prepayment penalties on mortgage rates and the likelihood of default. Consistent with our model, we show that the riskiest borrowers are the most likely to have prepayment penalties, with as many as 73 percent of subprime loans having prepayment penalties for borrowers with FICO scores under 620 compared with only about 50 percent of subprime loans with a FICO over 680.

Of course, many critics might argue that the riskiest borrowers are most susceptible to misunderstanding their mortgage and thus could be subject to increasing problems from being unable to prepay their mortgage. Yet the borrowers with the lowest FICO scores also receive the largest benefit from accepting prepayment penalties: their mortgage rates of 8.0 percent are about 0.7 percent lower than the 8.7 percent mortgage rate for similar borrowers with loans that are freely prepayable. By contrast, subprime borrowers with FICO scores over 680 obtain a mortgage rate on for purchase loans that is only 0.5 percent lower when they accept a prepayment penalty (7.5 instead of 8 percent).

Finally, we show that the riskiest borrowers with prepayment penalties also default at a much lower rate than borrowers without prepayment penalties. Borrowers with FICO scores less than 620 and no prepayment penalties default within 51 months at a 43 percent higher rate compared to borrowers with prepayment penalties (16.2% versus 11.3%). This difference disappears for the safer borrowers with FICO scores between 620 and 680, and over 680, who default about 8% and 4% of the time, respectively, whether or not their mortgage has a prepayment penalty. Our model suggests that the increase in default rate for the riskiest borrowers with no prepayment penalties may well be caused by the higher mortgage rates they pay.

In terms of the current policy debate, our competitive lending model and empirical work indicate that a ban on refinancing penalties for FRMs could be socially costly. If prepayment penalties are banned, risky borrowers could face mortgages with higher risk premiums leading to higher aggregate defaults for this group and/or lower home ownership rates. However, our results have an important limitation: they do not address the use of prepayment penalties in conjunction with ARMs such as 2-28 or 3-27 mortgages.

The paper is organized as follows. Section 2 presents the continuous-time setting of the model. Section 3 discusses the competitive mortgage lending with FRM contracts and no refinancing allowed. Section 4 studies the effect of refinancing on mortgage lending. Section 5 provides a computed example, while Section 6 discusses model extensions. Section 7 presents the empirical evidence. Section 8 concludes.

## 2 Setup

Time is continuous and infinite. There are two types of agents: the borrowers (homebuyers) and the lenders (financial institutions)<sup>4</sup>. Each of the lenders is risk neutral, has unlimited capital, and values a stochastic cumulative cash flow  $\{f_t\}$  as

$$E \left[ \int_0^{\infty} e^{-rt} df_t \right],$$

where  $r$  is the market interest rate at which the lender discounts cash flows.

The borrower values a stochastic cumulative consumption flow  $\{C_t\}$  and housing consumption flow  $\{C_t^H\}$  as

$$E \left[ \int_0^{\infty} e^{-\gamma t} (dC_t + dC_t^H) \right],$$

where  $dC_t \geq 0$  and  $dC_t^H \geq 0$ . The borrower's consumption at  $t$ ,  $dC_t \geq 0$ , represents the discretionary consumption of goods and services, which, among many other things, may include such items as restaurant dining, vacation trips, buying a new car, et cetera. The zero consumption ( $dC_t = 0$ ) means that the borrower consumes only necessities, such as food, medicine, transportation, and other goods and services essential for the household survival. We assume that housing consumption remains constant as long as the borrower stays in the same house.<sup>5</sup> We also assume that the borrower is more impatient than the lender, i.e.,  $\gamma > r$ , reflecting in our setting that the intertemporal marginal rate of substitution for a borrowing-constrained household is greater than those of a financial institution.

The borrower must use his income to first cover the necessary expenses  $\eta_t$  before spending on discretionary, "luxury", consumption. Let  $\bar{Y}_t \geq 0$  denote the borrower's total cumulative income up to time  $t$ . We will focus on the borrower's "excess" cumulative income,  $Y_t \equiv \bar{Y}_t - \eta_t$ , where  $\{\eta_t\}$  is a cumulative level of necessary consumption given by an exogenous stochastic process that incorporates shocks such as medical bills, auto repair costs, fluctuations of food and gasoline prices, and so on.<sup>6</sup> Therefore, the "excess" borrower's income  $Y_t$  represents a better measure of the borrower's ability to pay for a house than his total income. From now on we will refer to  $Y_t$  simply as the borrower's income.

We assume that a standard Brownian motion  $Z = \{Z_t, \mathcal{F}_t; 0 \leq t < \infty\}$  on  $(\Omega, \mathcal{F}, m)$  drives the borrower's income process, where  $\{\mathcal{F}_t; 0 \leq t < \infty\}$  is an augmented filtration generated by the Brownian motion. Ac-

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<sup>4</sup>Without loss of generality, we can think about the lender as a group of investors who maximize their combined payoff from the relationship with the borrower. How the investors divide proceeds among themselves is not relevant for the purpose of designing an optimal contract between the borrower and the investors.

<sup>5</sup>For simplicity, we do not consider a possibility that the borrower can make modifications that can either increase or decrease the quality of the house.

<sup>6</sup>This specification is similar in flavor to the one used by Ait-Sahalia, Parker and Yogo (2004) who propose a partial resolution of the equity premium puzzle by distinguishing between the consumption of basic goods and that of luxury goods. In their model, households are much more risk averse with respect to the consumption of basic goods, of which a certain amount is required in every period, which is consistent with the subsistence aspect of basic goods and the discretionary aspect of luxuries.

cordingly, the borrower's "excess" income up to time  $t$ , denoted by  $Y_t$ , evolves as

$$dY_t = \mu dt + \sigma dZ_t, \tag{1}$$

where  $\mu$  is the drift of the borrower's "excess" income and  $\sigma$  is the sensitivity of the borrower's income to its Brownian motion component. We assume that the lender knows  $\mu$  and  $\sigma$ , but does not know realizations of the borrower's excess income shocks  $Z_t$ . Thus, realizations of the borrower's income are not contractible. These assumptions are motivated by the observation that lenders use a variety of methods<sup>7</sup> to determine a type of the borrower (represented here by  $(\mu, \sigma)$  pair) before the loan is approved, but henceforth do not condition the terms of the contract on the realizations of the borrower's income, likely because the borrower's necessary spending shocks and possibly his total income as well are too costly or impossible to monitor.

The borrower maintains a savings account. The savings account balance  $S$  grows at the interest rate  $r$ . The borrower must maintain a non-negative balance in his account.

The borrower wants to buy a home at date  $t = 0$ .<sup>8</sup> For simplicity we assume that the borrower intends to live in this home forever. The ownership of the home generates him public and deterministic utility stream  $\frac{dC_t^H}{dt} = \theta$ , where we assume that  $\theta \leq \mu$ . The price  $P$  of the home is greater than the borrower's initial wealth  $S_0$ , i.e.,  $0 \leq S_0 < P$ .<sup>9</sup> Thus, the borrower must obtain funds from the lender to finance the house purchase.

Before the house purchase, the borrower and the lender sign a contract that will govern their relationship after the purchase is made. We assume that the borrower and the lender are sufficiently small so that their actions have no effect on macroeconomic variables such as the market interest rate.<sup>10</sup> In case of the loan foreclosure at time  $\tau$  the borrower receives the value of his outside option equal to  $A$ , which represents the borrower's continuation utility after the loss of the home plus the value of any savings he might have at the time of default  $S_\tau$ . The value  $A$  incorporates such factors as the consumption value  $\frac{\mu}{\gamma}$  of the borrower's expected future income, financial and intangible moving costs, losses with the damaged credit history, and the option to buy or rent another home in the future. The lender sells the repossessed house at a foreclosure auction and receives a payoff of  $L$ . We assume that  $L < P \leq \frac{\theta}{r}$  and  $A \leq \frac{\mu}{\gamma}$ , which makes the liquidation inefficient. We also make the following assumption, which insures that the default costs are not "too" small.

**Assumption 1** *Let  $r[P - (1 - \gamma)L] \geq \gamma\theta$ .*

The borrower with initial wealth  $S_0$  will decide to buy a house whenever the total utility he gets from

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<sup>7</sup>Like credit score, demographic variables and so on.

<sup>8</sup>To justify the initial purchase of the home, we assume that the borrower extracts more utility from the house when he owns it than when he rents it.

<sup>9</sup>The price  $P$  is considered as a macroeconomic variable, which is not affected by actions of the borrower and the lender. It is reasonable to expect that the home price  $P$  is increasing in its utility  $\theta$ , and the borrower optimizes over the set of available  $(\theta, P)$  pairs. This optimization is not considered in the paper. This clearly does not lead to a loss of generality, since our analysis applies to any  $(\theta, P)$  pair.

<sup>10</sup>In a general equilibrium framework, actions of mortgage lenders and homebuyers on the aggregate level can affect macroeconomic variables. However, as long as the economic agents on the individual level have no market power, they should regard macroeconomic variables as exogenous in an equilibrium.

homeownership is at least as big as the value  $R(S_0)$  he could get by not buying. The value  $R(S_0)$  represents the continuation utility of the borrower with initial wealth  $S_0$  if he decides not to purchase a house, which incorporates such factors as the consumption value  $\frac{\mu}{\gamma}$  of his expected future income, the value of savings  $S_0$ , and the option to buy or rent another home in the future. We assume that  $R(S_0) \geq \frac{\mu}{\gamma} + S_0$ , which implies that the outside value of prospective borrower, if he decides not to purchase a house, is at least as big as the sum of values of his initial wealth and the expected value of his "excess" disposable income.

### 3 Fixed rate mortgage with no refinancing

Before the house purchase, the borrower and the lender sign an exclusive contract that will govern their relationship after the purchase is made. The exclusivity of the relationship between the borrower and the lender can be guaranteed by the imposition of sufficiently high prepayment penalty that effectively acts as a commitment device that allows the borrower to credibly stay in the contract with the same lender until his default.

We assume that under the terms of the fixed-rate mortgage the borrower is required to make payments at the constant rate  $\bar{\phi}$ . If contract is signed, at time 0 the lender transfers the funds  $P$  needed to purchase a home to the borrower and commits to the terms of the contract. If the borrower fails to make the payment, the foreclosure procedure gets initiated and the borrower gets the value of  $A$  and the lender gets the value of  $L$ .

The borrower's total expected payoff from the mortgage with no refinancing at time zero is given by

$$\bar{U}_0 = E \left[ \int_0^\tau e^{-\gamma s} (\theta dt + dC_t) + e^{-\gamma \tau} A \right],$$

where  $\tau$  is the time when the borrower defaults on the mortgage. The lender's total discounted expected payments from the mortgage as of time zero are given by

$$\bar{V}_0 = E \left[ \int_0^\tau e^{-rs} \bar{\phi} dt + e^{-r\tau} L \right].$$

The borrower with initial wealth  $S_0$  will decide to buy a house whenever the total utility he gets under homeownership is at least as big as the value  $R(S_0)$  he could get by not buying, where we remember that  $R(S_0) \geq \frac{\mu}{\gamma} + S_0 \geq A + S_0$ . Given this, a net utility gain for the borrower from homeownership financed by the FRM with coupon  $\bar{\phi}$  and no refinancing allowed is bounded from above by  $\int_0^\tau e^{-\gamma t} (\theta - \bar{\phi}) dt$ . This implies the following Lemma.

**Lemma 1** *Any coupon payment  $\bar{\phi}$  in the FRM contract with no refinancing taken by the borrower satisfies  $\bar{\phi} \leq \theta$ .*

**Proof** Directly follows from the above discussion. ■

Since the borrower's income is stochastic while the mortgage payments are fixed, the borrower will need to save part of his income in order to be able to make mortgage payments in the future. Thus, for a given fixed-rate mortgage, the borrower's continuation payoff  $\bar{U}(S_t, \bar{\phi})$  is a function of the mortgage payments  $\bar{\phi}$  and balance  $S_t$  on the borrower's saving account. The borrower's savings evolve according to

$$dS_t = rS_t dt + dY_t - \bar{\phi} dt - dC_t. \quad (2)$$

The borrower chooses his consumption and savings strategy  $(C, S)$  to maximize his payoff given the mortgage contract. Obviously, the borrower will save only if  $\bar{U}'(S_t, \bar{\phi}) \geq 1$ , and consume only if  $\bar{U}'(S_t, \bar{\phi}) \leq 1$ , where prime denotes the derivative with respect to  $S$ . This implies existence of an upper bound  $S^1(\bar{\phi})$  on the amount the borrower will save. The following proposition formalizes this finding.

**Proposition 1** *Let  $\bar{U}(S, \bar{\phi})$  be a  $C^2$  function in  $S$  that solves*

$$\gamma \bar{U}(S, \bar{\phi}) = \theta + (rS + \mu - \bar{\phi}) \bar{U}'(S, \bar{\phi}) + \frac{1}{2} \sigma^2 \bar{U}''(S, \bar{\phi}) \text{ for } S \in [0, S^1(\bar{\phi})], \quad (3)$$

$$\bar{U}(0, \bar{\phi}) = A, \quad (4)$$

$$\bar{U}'(S, \bar{\phi}) = 1 \text{ for } S \geq S^1(\bar{\phi}), \quad (5)$$

$$\bar{U}''(S, \bar{\phi}) = 0 \text{ for } S \geq S^1(\bar{\phi}). \quad (6)$$

*The function  $\bar{U}(S_t, \bar{\phi})$  is concave and satisfies  $\bar{U}'(S, \bar{\phi}) > 1$  for  $S \in (0, S^1(\bar{\phi}))$ . For a given fixed-rate mortgage  $\bar{\phi} \leq \theta$  and savings  $S_t$  the borrower's continuation payoff equals  $\bar{U}(S_t, \bar{\phi})$ . The optimal strategy for the borrower is to have no luxury consumption when  $S_t \in [0, S^1(\bar{\phi})]$ , and to consume  $S_t - S^1(\bar{\phi})$  immediately when  $S_t > S^1(\bar{\phi})$ .*

**Proof** In the Appendix. ■

**Lemma 2** *The borrower's continuation payoff  $\bar{U}(S, \bar{\phi})$  is decreasing in the repayment rate  $\bar{\phi}$ .*

**Proof** We remember that  $\bar{U}(S, \bar{\phi})$  represents the total utility the borrower with initial savings of  $S$  gets from the mortgage with a coupon payment of  $\bar{\phi}$ . Therefore, the more the borrower is asked to pay the lower will be her utility under the FRM contract. ■

Given a choice of  $\bar{\phi}$  the lender's total expected payments from the FRM mortgage with no refinancing allowed given to a borrower with initial wealth (savings) of  $S_0$ , as of time 0, are equal to

$$\begin{aligned} V_0(S_0, \bar{\phi}) &= E \left[ \int_0^{\tau(S_0, \bar{\phi})} e^{-rt} \bar{\phi} dt + e^{-r\tau(S_0, \bar{\phi})} L \mid \mathcal{F}_0 \right], \\ &= \frac{\bar{\phi}}{r} - E \left[ e^{-r\tau(S_0, \bar{\phi})} \left( \frac{\bar{\phi}}{r} - L \right) \mid \mathcal{F}_0 \right]. \end{aligned}$$

In the above  $\tau(S_0, \bar{\phi})$  is the default time of the borrower implied by his optimal choice of consumption and savings characterized in Proposition 1, given his initial wealth  $S_0$  and the required mortgage payments of  $\bar{\phi}$ .

We assume that the lending market is competitive. Therefore, given Lemma 2, the lender chooses the smallest  $\bar{\phi}$  so that he breaks even. This leads us to the following definition.

**Definition 1** *The competitive mortgage repayment rate on the FRM with no refinancing for the borrower with initial level of wealth (savings)  $S_0$  satisfies*

$$\bar{\phi}^*(S_0) = \{\inf \bar{\phi} \geq 0 : V_0(S_0, \bar{\phi}) = P\},$$

and the implied mortgage premium is given by

$$\rho^*(S_0) = \frac{\bar{\phi}^*(S_0)}{P} - r.$$

The borrower takes the loan whenever

$$\bar{U}(S_0, \bar{\phi}^*(S_0)) \geq R(S_0).$$

Let  $\underline{S}$  be the minimum level of wealth such that the borrower takes the loan and the lender breaks even. Let  $\bar{S}$  be the minimum initial level of wealth of the most creditworthy borrower (the borrower with the smallest  $\bar{\phi}^*$ ).

**Proposition 2** *The competitive mortgage repayment rate on the FRM with no refinancing,  $\bar{\phi}^*$ , is strictly decreasing in the borrower's initial savings on  $[\underline{S}, \bar{S}]$ . Moreover*

$$\bar{S} = \inf_{S \geq \underline{S}} \{S^1(\bar{\phi}^*(S))\}$$

**Proof** In the Appendix. ■

The above proposition is intuitive. The mortgage premium compensates the lender for the loss due to default. The lower is the initial wealth (savings) of the borrower, the higher is the probability of his default (the lower is the borrower's creditworthiness), and so the larger is the premium charged on the loan. In the above proposition  $\bar{S}$  represents a wealth level of the most creditworthy borrower who obtains the lowest possible mortgage rate  $\bar{\phi}^*(\bar{S})$ .

## 4 Fixed rate mortgage with refinancing

In this section, we allow for refinancing of mortgage loans. The borrower can refinance the loan each time his creditworthiness sufficiently improves. The borrower sticks to the existing loan when his creditworthiness

deteriorates, as then refinancing would imply a higher interest rate premium on the mortgage and thus would make him worse-off.

Given the borrower's type, a mortgage coupon of  $\bar{\phi}$  and a loan amount of  $P$ , the borrower's creditworthiness is measured by his credit score that reflects the chances of default on the mortgage. We assume that the borrower has no debt other than the mortgage itself. Then the borrower's creditworthiness (credit score) increases in his wealth level  $S$ .

The borrower can refinance the loan when his credit score improves by a certain amount<sup>11</sup>. This is represented by an increasing sequence of savings cutoffs  $\{S^i\}_{i=1}^K$ , where  $S^i = 0$  and  $S^K = \bar{S}$ , where  $\bar{S}$  is defined in Proposition 2. Consider a borrower with initial wealth level of  $S_0$ . Let

$$N_{S_0} = \#\{S > S_0 : S \in \{S^i\}_{i=1}^K\}.$$

Define a sequence  $\{S_{S_0}^n\}_{n=0}^{N_{S_0}}$  as

$$\begin{aligned} S_{S_0}^0 &= S_0, \text{ and if } N_{S_0} > 0 : \\ \{S_{S_0}^n\} &= \min\{S > S_{S_0}^{n-1} : S \in \{S^i\}_{i=1}^K\}, \text{ for } n = 1, \dots, N_{S_0}. \end{aligned}$$

Then the borrower with initial wealth  $S_0$ , can refinance the loan each time his wealth reaches for the first time one of the wealth levels  $\{S_{S_0}^n\}_{n=1}^{N_{S_0}}$ .

**Proposition 3** *Let  $S_0$  be an initial wealth of the borrower.*

(i) *If  $S_0 \geq \bar{S}$ , the borrower's continuation payoff and the mortgage coupon under FRM with refinancing and competitive lending are equal to those with no refinancing.*

(ii) *If  $S_0 < \bar{S}$  then  $N_{S_0} > 0$ . Under the competitive lending market it is optimal for the borrower to refinance the loan whenever his wealth reaches for the first time one of the wealth levels  $\{S_{S_0}^n\}_{n=1}^{N_{S_0}}$ . The borrower's continuation payoff after  $n$ th refinancing,  $U^n(S, \phi^n)$  for  $n = 0, \dots, N_{S_0}$ , is given by a concave twice continuously differentiable function that solves for  $n = 0, \dots, N - 1$  :*

$$\gamma U^n(S, \phi^n) = \theta + (rS + \mu - \phi^n)(U^n)'(S, \phi^n) + \frac{1}{2}\sigma^2(U^n)''(S, \phi^n) \text{ for } S \in [0, S_{S_0}^{n+1}],$$

$$U^n(0, \phi^n) = A,$$

$$U^n(S_{S_0}^n, \phi^n) = U^{n+1}(S_{S_0}^n, \phi^{n+1}),$$

and  $U^{N_{S_0}}(S, \phi^{N_{S_0}}) = \bar{U}(S, \phi^{N_{S_0}})$  for all  $S \geq 0$ . The competitive market coupon payments  $\{\bar{\phi}^n\}_{n=0}^N$  are

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<sup>11</sup>This assumption is justified by a discrete nature of the credit scoring technology.

given by

$$\begin{aligned}\bar{\phi}^n &= \{\inf \bar{\phi} \geq 0 : V_0^n(S_{S_0}^n, \bar{\phi}) = P\}, \text{ for } n = 0, \dots, N_{S_0} - 1, \\ \bar{\phi}^{N_{S_0}} &= \bar{\phi}^*(\bar{S}),\end{aligned}$$

where for  $n = 0, \dots, N_{S_0} - 1$ :

$$V_0^n(S_{S_0}^n, \bar{\phi}) = E \left[ \int_0^{\min(\tau, \tau^n)} e^{-rt} \bar{\phi} dt + e^{-r \min(\tau, \tau^n)} (L + 1_{\tau^n < \tau} (P - L)) \right],$$

and where

$$\begin{aligned}\tau &= \inf\{t > 0 : S_t = 0 \mid S_0 = S_{S_0}^n\}, \\ \tau^n &= \inf\{t > 0 : S_t = S_{S_0}^{n+1} \mid S_0 = S_{S_0}^n\}, \\ dS_t &= rS_t dt + \mu dt + \sigma dZ_t - \bar{\phi} dt \text{ on } [0, S_{S_0}^{n+1}].\end{aligned}$$

The borrower takes the loan whenever  $U^0(S_0, \phi^0) \geq R(S_0)$ .

**Definition 2** Let  $\underline{S}^{ref}$  be the minimum wealth (savings) of the borrower such that the borrower takes the FRM loan with refinancing and the lender breaks even.

**Proposition 4** If  $S_0 \in [\underline{S}^{ref}, \bar{S}]$ , the expected utility for the borrower under the competitive lending is greater under the loan without refinancing (with prepayment penalty):

$$\bar{U}(S_0, \bar{\phi}^*) > U^0(S_0, \phi^0),$$

and

$$\phi^0 > \bar{\phi}^*(S_0).$$

Moreover,

$$\frac{d(\bar{U}(S_0, \bar{\phi}^*) - U^0(S_0, \phi^0))}{dS_0} < 0,$$

and

$$\underline{S}^{ref} > \underline{S}.$$

The above proposition shows that the borrowers are worse off under the FRM contract when refinancing is allowed. Moreover, the more so the worse is their creditworthiness (their initial wealth). Allowing mortgage refinancing also leads to the exclusion from credit of more risky borrowers compared to the contracts with no refinancing (FRM contracts with sufficiently high refinancing penalty). As homeownership is assumed to generate positive utility gains for the borrowers, this exclusion from credit would also lead to lower utility

for those who would otherwise qualify for credit if refinancing were not allowed. Consequently, imposing a prepayment penalty is Pareto improving in this environment.

To illustrate this point consider a group of ex-ante identical borrowers obtaining FRM loans to purchase identical homes. Suppose that their initial wealth level (initial creditworthiness) is such that they would qualify for loan under both regimes (with and without refinancing allowed). As the borrowers are ex-ante identical, they will be charged the same premia on their loans. We recall that the mortgage premium compensates the lender for the expected losses due to defaults. Under the FRM contract without prepayment penalty, those borrowers who become more creditworthy over time would refinance to obtain cheaper premia on their loans, leaving less creditworthy behind. The rational lender would anticipate and compensate for this by charging a uniformly higher premium on the loans compared to the contract with prepayment penalty. At first one could think that allowing for refinancing is welfare-neutral for those borrowers who would qualify for credit under both regimes. On one hand, ex-post less creditworthy borrowers (those who received bad shocks to their financial position) would be worse off compared to the contract with a prepayment penalty as they would have to pay a higher premium. On the other hand, those borrowers whose creditworthiness would sufficiently improve ex-post (those who received positive shocks to their financial position) would refinance to lower premia and thus be better off compared to the contract with a prepayment penalty. The Proposition 4 shows that these expected gains if the borrower's creditworthiness improves are not sufficient to offset the expected losses when the borrower's creditworthiness deteriorates. Why is it so? Charging a higher premium makes the borrowers more likely to default, and the likelihood of default is more sensitive to premia for those who are less creditworthy. Consequently, the decrease in likelihood of default due to lower premia of those who refinance is not sufficient to offset an increase in the likelihood of default due to higher premia paid by those whose creditworthiness deteriorates ex-post and so they cannot refinance. As a result, refinancing increases the likelihood of default in a given pool of the borrowers. As default is costly, this leads to an additional welfare loss which under competitive mortgage lending is passed on the borrowers. Moreover, a required increase in premium to compensate for refinancing leads to an exclusion from credit of a riskier class of borrowers who would otherwise qualify for loans if refinancing were not allowed. As homeownership is assumed to generate positive utility, this effect also reduces welfare.

A sufficiently high prepayment penalty allows the borrowers to credibly commit to staying with the same lender. As a result, lenders can charge lower premia due to potential to cross subsidize. This cross-subsidization effectively provides a partial insurance against reclassification risk; ex-post more creditworthy borrowers end up subsidizing those whose creditworthiness has deteriorated. This lowers the overall likelihood of socially costly default. Consequently, a prepayment penalty is Pareto improving in this environment.

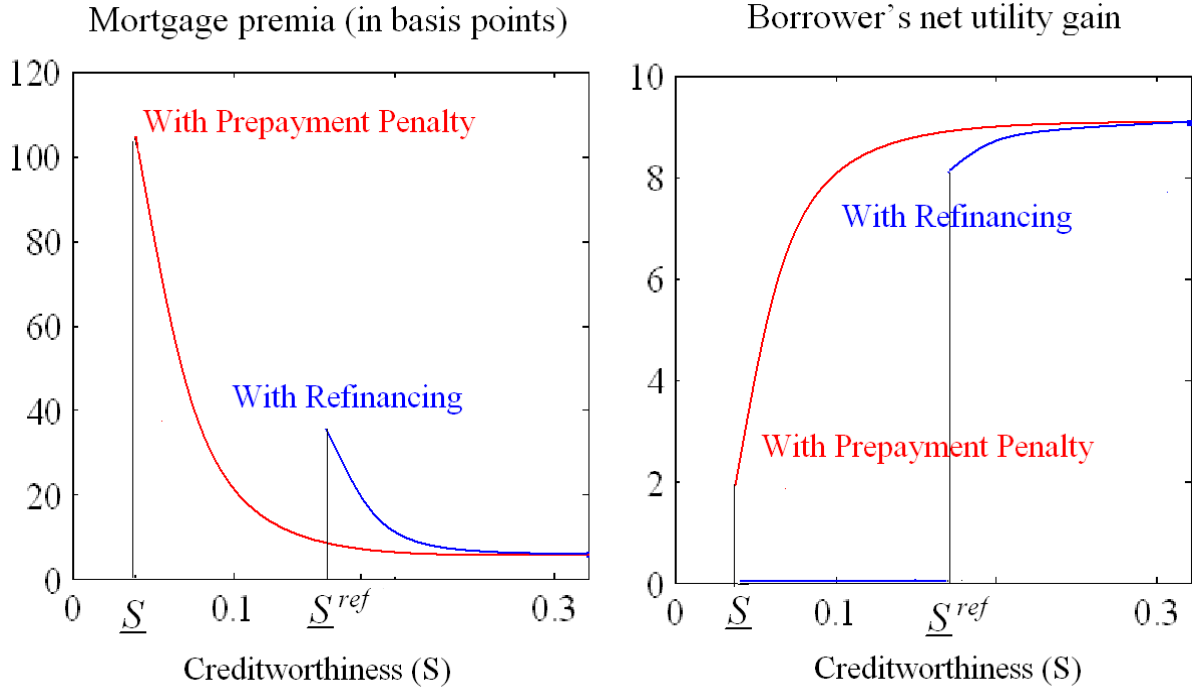


Figure 1: Mortgage premia and the borrower's net utility gain from homeownership.

## 5 Insights from a computed example

In this section we illustrate the features of the competitive FRM mortgage lending with and without refinancing in a parametrized example. Table 1 shows the parameters of the model. The refinancing wealth cutoffs are set as  $\{S^i\}_{i=1}^{K-1} = \{0.02i\}_{i=1}^{K-1}$ , with  $S^K = \bar{S} = 0.33$ .

Table 1. Parameters of the model

Interest rate	Borrower's discount rate	Income process		Utility flow from homeownership	Liquidation and reservation values			Home price (loan amount)
$r$	$\gamma$	$\mu$	$\sigma$	$\theta$	$A$	$L$	$R(S_0)$	$P$
3%	5%	1	0.2	1	20	14	$\frac{\mu}{\gamma} + S_0$	18

The left-hand side of Figure 1 shows the FRM mortgage premia under a competitive lending market without refinancing (red curve) and with refinancing (blue curve) allowed as a function of the borrower's creditworthiness (initial wealth level). As before no refinancing is justified by an imposition of sufficiently high prepayment penalty.

We remember that the borrower’s credit score increases in his wealth level  $S$  (the likelihood of default decreases with the wealth level). The vertical black lines show the wealth cutoffs such that the borrower with wealth below it does not qualify for a loan. We observe that both with and without refinancing allowed mortgage premia are decreasing in the wealth level (credit score) reflecting a lower likelihood of loss due to default. The mortgage premia with refinancing are larger compared to those without refinancing, and the more so the riskier is the borrower (the lower is his wealth level). We also observe that allowing for refinancing leads to a significant exclusion from the lending market of riskier types (those with lower wealth levels). Interestingly, it is the lender’s participation constraint that dictates the exclusion from credit. In the regime with refinancing, the riskiest borrowers served (those with initial wealth of  $\underline{S}^{ref}$ ) has a large net positive utility gain from homeownership; however, the lenders cannot break even on those with lower wealth. This is because increasing the mortgage premium does not necessarily increase proceeds from the borrower, as higher premium implies higher likelihood of default.

The right-hand side of Figure 1 shows the borrower’s net utility gain from homeownership financed by the FRM mortgage without (red curve) and with (blue curve) refinancing allowed as a function of the borrower’s initial wealth level. We observe that the borrower’s net utility gain is lower when refinancing is allowed, the more so the riskier is the borrower. However, a large efficiency loss from refinancing in this example comes from the exclusion from the lending market of riskier types who cannot enjoy the benefits of homeownership. Note that under this parametrization one would see more high cost (high premia) loans and more defaults if prepayment penalties are used, just because more risky borrowers would be able to qualify for a loan.

## 6 Extensions

In this section, we discuss possible extensions of our model. We argue that, even if we take into account additional factors, the main finding of the model still holds: prepayment penalties are beneficial for borrowers.

This result is based on the following argument. In order to break even, the lender has to make money on the borrowers who receive positive wealth shocks, since the lender is likely to lose money on the borrowers who receive negative wealth shocks. However, in the absence of prepayment penalties, the lender’s ability to make profit is more limited, since the borrowers with improved creditworthiness are able to refinance their mortgages and get a lower mortgage premium. Hence, the lender has to either charge a higher mortgage premium on mortgages without prepayment penalties, or avoid serving risky borrowers at all. We maintain that the same argument is valid in more general settings discussed below.

### 6.1 Stochastic House Prices

So far we have considered a time-homogeneous setting in which agents are infinitely lived and the borrower’s average income and the liquidation and reservation values do not change over time. In a stochastic house price environment, an increase in house price would increase the borrower’s creditworthiness (due to an increase

in the value of collateral and his total wealth) and thus provide an additional incentive to refinance if this option is allowed. Similarly a decline in house price would lower the borrower's creditworthiness. Therefore, our argument supporting refinancing penalties would be stronger in a stochastic house price environment due to an additional source of variation in ex-post credit quality of the borrowers. In fact in our empirical work we will use house price shocks as a proxy for ex-post wealth shocks.

One of the implications of our model is that the borrower does not default as long as his savings remains positive. It is possible, however, that the borrower may strategically default even when he is still able to make mortgage payments. A strategic default could be triggered, for example, by a sufficient decline in the home value. When the borrower's debt exceeds the value of his home, he may want to default on the existing mortgage and buy a new home at a lower price.

Taking into account the possibility of strategic default would only strengthen our argument. Indeed, allowing for strategic default means that borrowers who receive a negative wealth shock would default sooner rather than later. This results in bigger losses for the lender when the borrower receives a negative wealth shock, but it does not increase the lender's profit when the borrower receives a positive wealth shock.

## **6.2 Endogenous Refinancing Grid**

For simplicity, we assumed that refinancing happens on the exogenously determined refinancing grid. A more realistic assumption is that refinancing happens whenever it improves the borrower's continuation utility by a certain amount. This determines an endogenous refinancing grid. Since our results hold for any grid, this does not change the predictions of our model. However, it generates a new prediction. Since the benefits of refinancing are higher for riskier borrowers, these borrowers are going to refinance their mortgages more often in response to positive wealth shocks. We test this prediction using house price shocks as a proxy for ex-post wealth shocks.

## **6.3 Mobility and cash out motives to prepay**

The findings in our model are predicated on the assumption that borrowers have no reason to prepay their mortgage except to receive a lower mortgage premium. Of course, this analysis ignores the three other likely reasons that borrowers prepay mortgages: lower interest rates, cash-out refinancings, or the sale of a house to move to another location. While interest rate changes present a complication that is beyond the scope of our current model, we believe that prepayments based on mobility and cash out motives should not unduly bias our findings.

Even in our sample at the lowest interest rate time period, about 1 in 5 of highest quality prime borrowers prepay their mortgage within 51 months after origination. Borrowers who recognize a high likelihood that they might move in the near future would surely prefer to avoid prepayment penalties. Such moves may take place for employment or family reasons. Either way, borrowers who are likely to move receive a countervailing

benefit of avoiding prepayment penalties that is not in our model. What would appear in our model to be an inefficient prepayment might instead be an efficiency enhancing move. Prepayment penalties that restrict mobility might well have a negative welfare effect.

It is worth noting that financial institutions can take several steps to mitigate the negative effects of prepayment penalties on mobility. After all, the vast majority of developed countries have mortgages that commonly contain prepayment penalties. In Canada, for example, a borrower must pay the prepayment penalty regardless of the reason they payoff their mortgage early. However, a borrower who takes out a mortgage on a new house from the same lender as his original property will typically have the prepayment penalty refunded. This policy is similar to allowing the borrower to take the mortgage to a new house as long the value of the new house is at least as high as the previous property (a so-called "transportable" mortgage). A second possibility that was common in the US in the late 1970s and is still commonly used for commercial mortgages is assumable mortgages. With an assumable mortgage, the owner can always sell the property subject to the existing mortgage, thus avoiding the prepayment penalty. If the mortgage rate is sufficiently unattractive, the seller would presumably have to lower the sales price. However, this ex-post cost to the borrower compensates for a lower mortgage rate ex-ante on a mortgage with a prepayment penalty.

If we assume that transportable and assumable mortgages are not feasible, it is still straightforward to incorporate moving costs in our model. We must simply assume that borrowers face some unmodeled cost associated with prepayment penalties (the cost associated with reduced mobility). For the highest credit quality borrowers where reclassification risk is extremely low, the benefit of a prepayment penalty in terms of a lower rate premium is minimal. These borrowers already receive the lowest available mortgage rate. Thus even a small benefit of moving is likely enough to tip them in favor of choosing a fully prepayable mortgage. Thus we would expect that the highest quality borrowers will almost surely want to avoid prepayment penalties. By contrast, the low credit quality borrowers receive a very large discount for accepting a prepayment penalty. For these borrowers, the likelihood of mobility must be very high for them to choose a fully prepayable mortgage. Such risky borrowers with a strong likelihood of moving might well be better off as renters.

To develop these ideas further let's assume that a borrower of a given type attaches a certain value to mobility/flexibility. Let  $f(\mu, \sigma, \theta, \gamma, P, S_0)$  be a distribution of the utility gains from mobility for a borrower of a type  $(\mu, \sigma, \theta, \gamma, P, S_0)$ . Then a given type with a realization  $U^m$  of utility value from mobility would choose a mortgage with prepayment penalty whenever

$$\bar{U}(S_0, \bar{\phi}^*) > U^0(S_0, \phi^0) + U^m$$

where the dependence of the values  $\bar{U}$  and  $U^0$  on  $(\mu, \sigma, \theta, \gamma, P)$  is implicit. Note that when  $S_0 \geq \underline{S}^{ref}$ , not choosing a mortgage with refinancing penalty means obtaining a mortgage without refinancing penalty. However, for  $\underline{S} \leq S_0 < \underline{S}^{ref}$  the borrower who values mobility sufficiently will choose to remain a renter.



## 6.4 Finite Prepayment Penalties

We have considered a fixed rate mortgage with no refinancing, which has, by definition, an infinitely high prepayment penalty, and a fixed rate mortgage that has no prepayment penalty. One of the predictions of our model is that the prepayment penalty leads to the interest rate reduction. Since the prepayment is banned completely, the interest rate reduction cannot be attributed to the possibility that the borrower pays the prepayment penalty to the lender thus compensating him for the lower interest rate. Instead, we show that the interest rate reduction is welfare improving.

In practice, prepayment penalties are finite and borrowers sometimes choose to prepay their mortgages despite the prepayment penalties. It is easy to see, however, that considering a finite prepayment penalty will not change the predictions of our model. With a finite prepayment penalty, refinancing, although possible, is not going to happen often. Moreover, even if the borrower's creditworthiness improves sufficiently so that refinancing benefits him, the lender will be partially compensated with the prepayment penalty. Thus, by imposing a finite prepayment penalty, the lender will be able to extend credit to more risky borrowers and lower the mortgage premia.

## 6.5 Adverse selection

A second potential concern with our model is the assumption that the lender accurately observes the credit quality of the borrower. That is, lenders know as much about the likelihood of a default using observable information as the borrower. We believe this assumption is reasonable given that the lender can observe income, occupation, credit history, and the loan-to-value ratio of the property. Underwriting experience allows lenders to determine the probability distribution of defaults. Lenders are also likely to be better informed than borrowers about the distribution of future house price changes. (We recognize the poor underwriting in the mortgage industry makes these assumptions less plausible for the 2005 to 2007 time period. This provides an additional advantage of choosing mortgages originated in June, 2003 prior to the worst subprime lending excesses.)

We also ignored the possibility that menu of contracts with and without refinancing penalties could play a role in screening borrowers based on their ex-ante knowledge concerning likelihood of moving<sup>12</sup>. This could be the case if borrowers were better informed than lenders about the ex-ante riskiness of their income, wealth, or the likelihood of their mobility shocks. The authors considered several potential adverse selection stories. None of those stories, however, could generate predictions consistent with the predictions of our model.

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<sup>12</sup>See for example Brueckner (1994), Chari and Jagannathan (1989), LeRoy (1996).

## 6.6 Non competitive lending, more complex contracts

Our results were derived in a competitive mortgage lending market with FRM contracts. We conjecture that the inefficiency of refinancing we identified would also apply to others models of competition and environments with more complex contracts<sup>13</sup>, as in general exclusivity would increase total surplus there as well. The relative bargaining power of the borrower would dictate how much of an additional surplus generated by exclusive relationship with the lender he would be allocated.

## 6.7 Stochastic interest rate

The prepayment penalties should remain attractive in a model with stochastic interest rates as the basic intuition behind the efficiency of borrowers cross-subsidizing each other to hedge against future risks will be valid there as well.<sup>14</sup>

## 6.8 General risk aversion

For the sake of tractability, we assumed risk-neutrality of the borrower with respect to luxury consumption. A more general form of risk-aversion on the borrower's side would likely strengthen our results as the risk averse borrower would value insurance against costly default, which the lenders can better provide employing refinancing penalties.

## 6.9 Endogenous downpayment

Finally, for tractability, we assume a constant (zero) downpayment by the borrower. In the health and life insurance context frontloading contributions can help alleviate the problem of insurance of reclassification risk.<sup>15</sup> In the lending context the beneficial role of frontloading through downpayment or points would be much weaker. Those who can spend a lot on points or higher downpayments are likely already good risks, so placing a large downpayment might not benefit them a lot in terms of overcoming the reclassification risk<sup>16</sup> unless downpayments are the only way to credibly signal that they are good risk borrowers. Evidence from other countries where credit reporting is less accurate and housing markets are more opaque suggest that the screening role of large downpayments might be more important when information is less verifiable.

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<sup>13</sup>See Piskorski and Tchisty (2007, 2008) who derive the optimal mortgage contract in similar environments.

<sup>14</sup>In a stochastic interest rate environment there could be a welfare benefit in allowing the mortgage contract to depend on the risk-free interest rate. Piskorski and Tchisty (2007) derive an optimal mortgage in a similar environment and show that it resembles the option ARM where full exclusivity (prepayment penalty) is still optimal.

<sup>15</sup>See for example Hendel and Lizzeri (2003).

<sup>16</sup>This is an important difference with health or life insurance context where the agent's ability to frontload an insurance premium does not necessarily indicate lower likelihood of death or serious illness.

## 7 Empirical Evidence

The model in the previous section makes a number of important predictions that we now examine using data on recently originated fixed rate mortgages. As suggested above, we use house price changes as a proxy for ex-post credit shocks. First, we limit the sample to borrowers with mortgages that do not have prepayment penalties in order to examine the extent of reclassification risk. Then, we compare attributes of loans with and without prepayment penalties.

In examining the key predictions from our model, we consider the leading alternative hypotheses explaining why loans with prepayment penalties are more prevalent among high risk borrowers and why risky borrowers who receive positive credit shocks may be more likely to pay off their loans early.

In recent times, commentators and politicians have argued that financially unsophisticated borrowers may not be able to fully understand or value prepayment penalties. Recent research by Bucks and Pence (2006) supports the case that borrowers often do not understand all the features of their mortgages, although the authors do not specifically address prepayment penalties. The authors find that less educated and minority borrowers are quite likely to mis-report the likelihood and magnitude of potential interest rate adjustments in ARMs. However, borrowers appear to accurately report purchase price and their current payments, so borrowers are aware of at least some key pieces of mortgage and housing information. Mortgage disclosures in the US are not designed to fully address prepayment penalties, which were relatively rare until the development of the subprime mortgage market. As such, one might believe that prepayment penalties are more prevalent among the highest risk borrowers because these borrowers may not fully understand the implications of prepayment limitations, especially if such borrowers are also more likely to be minorities or less educated borrowers. If borrowers that accept prepayment penalties are unsophisticated or naive, we might expect that such borrowers might pay higher rates and/or default at a higher rate, holding their ex-ante risk constant.

Of course, borrowers also might refinance their property to obtain cash out of their property in response to price increases, either to increase consumption directly (a wealth effect) or to pay off higher cost debt such as credit cards or student loans. Unfortunately, data on mortgages typically do not list the type of prepayments, such as a property sale, a straight refinancing (for a lower rate), or a cash-back refinancing. Such cash back refinancings are almost surely more likely for the lowest credit quality borrowers. Such low credit quality borrowers are more likely to have higher cost debt so that they could disproportionately benefit from debt consolidation through a new mortgage. Of course, cash constrained sellers can still easily and cheaply use a home equity line of credit to obtain cash as opposed to refinancing the mortgage. Home equity lines of credit were widely available over our sample period.

In terms of our arguments, from a lender's perspective, it does not matter why a borrower prepays their mortgage as long as the lender faces the likelihood that borrowers with improved risk profile are more likely to prepay. We may have a difficult time disentangling cash-out refinancing motives from the desire to obtain

a lower mortgage spread. However, wealth effects of a cash-out refinancing are quite similar to the motive of refinancing to obtain a lower mortgage rate. Both decisions involve using the house to obtain lower cost capital for new expenditures, or to pay off debt (the mortgage).

We face one other key empirical challenge in examining our model. Up to now, we have not considered the possibility that interest rates change over time. In fact, our model assumes fixed interest rates, so that wealth shocks are the only reason that borrowers prepay their mortgage. In reality, interest rate declines are typically the leading reason that borrowers refinance.

To address this issue, we limit our sample to a single month, June, 2003. As shown in Figure 3, mortgage rates in this month hit their lowest rate in the last decade. Thus any mortgages originated in this month should be the lowest rate available, so borrowers choosing to refinance must be doing so for other reasons besides interest rate declines.

## 7.1 Empirical predictions

Below we develop predictions that are consistent with our model but also help to distinguish between reclassification risk and the alternative of the limited ability of risky borrowers to understand their loans. These predictions include:

*Borrowers who receive (unexpected) positive credit shocks are more likely to prepay their mortgages.* We examine mortgages without prepayment penalties to see how likely these mortgages are to prepay in response to house price shocks.

*The sensitivity of prepayment risk to a positive credit shock is larger for lower credit quality households.* While the alternative model speaks to this prediction directly, our credit model strongly suggests that the worst credit risk borrowers benefit the most from positive credit shocks and should be most likely to prepay their mortgages.

*Prepayment penalties should be most prevalent for the riskiest borrowers.* According to our model, the riskiest borrowers should be the most likely to choose prepayment restricted mortgages but also to receive the largest benefit (biggest spread differential) for accepting a prepayment penalty in a mortgage.

*Borrowers with prepayment penalties obtain rates that are lower than similar borrowers with fully prepayable mortgages, with the largest reductions going to the riskiest borrowers.* That risky borrowers should receive the largest benefit in the form of cheaper mortgage rates in return for accepting a mortgage with prepayment restrictions counteracts the view that such borrowers may simply be fooled into accepting a prepayment penalty without receiving something in return.

*Low credit quality borrowers without prepayment penalties should default at a higher rate than borrowers with prepayment penalties.* According to our model, borrowers who accept a prepayment penalty will pay a lower mortgage rate and thus be better able to respond to negative shocks without defaulting. In this regard, prepayment penalties help reduce the number of welfare reducing defaults and foreclosures.

## 7.2 Data Summary

Our primary data comes from LoanPerformance (LP), a subsidiary of First American CoreLogic. LP provides loan-level data on a large number of securitized mortgages. Mayer and Pence (2008) suggest that the LP data appear relatively representative of the universe of high-cost risky loans, with the exception that refinancings appear to be somewhat over-represented in LP. LP collects its data at two different times. First, LP collects data on contract terms at the time of origination. In addition, LP also collects data on whether or not the loan has paid off or become delinquent from servicers throughout the life of the mortgage. We combine these sets of information to create a monthly, loan level, panel dataset that includes both the characteristics of a loan at origination as well as its payment history.

Within the LP database, we consider only loans with the following characteristics:

- a. 30 year term
- b. Fixed interest rate (we do not consider ARMs or hybrid mortgages)
- c. Known prepayment penalty status (some loans have the prepayment field missing)
- d. Located in an MSA with house price index (HPI) data
- e. Collateralized by an owner-occupied home

We collect HPI data from the Office of Federal Housing Enterprise Oversight. The data are reported at the MSA level and are mapped onto the LP data using a zip code to MSA correspondence. The HPI index is normalized to reflect real dollars. Table 2 defines all the variables we use in our analysis.

We consider three subsets of the LP database: prime, Alt A and subprime. Loans are classified by the type of pool they belong to using definitions that are self-reported by the issuer of the mortgage-backed security. Prime MBS are backed by high-quality mortgages (that is, mortgages for borrowers with relatively low loan-to value ratios and with very good credit scores), whose initial balance typically exceeds the maximum limits for participation by government-sponsored entities Fannie Mae and Freddie Mac. Alt-A pools have borrowers with poorer FICO scores and LTV's than prime borrowers, have no mortgage limit, most often involve investor loans, and many times have little or no income verification. Subprime pools usually have the riskiest borrowers, with the lowest FICO scores, highest loan-to-value ratios, and with no mortgage limit.

We begin with a sample of about 28,000 prime jumbo mortgages, 17,000 Alt-A loans, and over 50,000 subprime mortgages. Of those, slightly less than half are for FRMs, with most of the attrition coming from the subprime pools, where hybrid ARMs are the predominant mortgage type. We remove from this sample loans according to our restrictions listed above. This leaves us with a sample of 23,633 subprime FRMs (of which 8,715 carry no prepayment penalty), and also make use a sample of 10,630 prime FRMs and 9,601 Alt-A FRMs that carry no prepayment penalties in order to investigate the refinancing propensity of the borrowers.

Table 3 reports summary statistics for the securitized loans in our sample that carry no prepayment penalties. Since loans are classified by the pool of mortgages, some loans in the relatively risky subprime

category could be safer than mortgages in Alt-A pools. In general, loans in Prime pools are quite safe along all dimensions, with a mean combined loan-to-value (CLTV) ratio of 64.7 percent and a FICO score of 740. Almost two-thirds of the loans are fully documented. Alt-A pools contain somewhat riskier loans with an average CLTV of 77.9 percent and a mean FICO of 710. Credit quality deteriorates most markedly in subprime pools, with a mean CLTV of 89 percent and a FICO of only 658. However, contrary to popular perception, more than 60 percent of the subprime loans in our sample are fully documented. These statistics suggest variation across multiple risk factors that complicate our analysis. Among the categories of loans, subprime loans prepay in the first 51 months at much higher rates (68%) compared to Alt-A (55%) and prime loans (32%), suggesting the possible heightened sensitivity of subprime loan pools to reclassification risk.

Houses also experience quite different rates of house price appreciation depending on which metropolitan area they are located in. The mean quarterly real appreciation rate was 1.7 percent (about 7 percent annualized), reflecting the strong growth of house prices over our sample period. Thus we would expect relatively few defaults, as borrowers who get into financial trouble can respond by paying off their mortgage by selling their house, often at a profit. However there is wide dispersion in house price growth rates. The highest appreciation markets experiencing appreciation rates as high as 3.3%—almost 14 percent per year for more than 4 years. Slightly less than ten percent of markets saw negative real appreciation rates over this time period.

Table 5 reports summary statistics for all securitized subprime loans in our sample and also separately for purchase loans. With the possibility of refinancings for cash-out motives that are not strictly considered in our model, many specifications are also run separately for purchase mortgages. Defaults (defined as loans that become at least 90-days delinquent or are in the foreclosure process) are much more concentrated in the subprime pools, representing about 10 percent of all loans. By contrast, Alt-A pools had only a 3.3 percent default rate, while prime loans had a miniscule 0.3 percent default rate.

### 7.3 Empirical results

We begin by exploring our first hypothesis that the borrowers who receive positive credit shocks are more likely to prepay their mortgages. Table 4 presents the set of regressions that explore the prediction that higher house prices spur increased prepayments, so that loans in locations with below-average house price appreciation become disproportionately represented in a pool of loans. This is the essence of reclassification risk. We use a logit specification with a dependent variable that equals 1 if a loan in a given category pays off and zero otherwise. The independent variable of interest is the annualized rate of house price price appreciation. The specifications include a variety of control variables that are commonly associated with loan payoff and risk, including the coupon rate on the mortgage, the borrower’s FICO score, cumulative loan-to-value ratio, whether the loan was a refinancing or cash-out refinancing, and whether there was an interest

rate bydown. We run separate regressions for prime, Alt-A and subprime pools. For ease of interpretation, coefficients in all tables reflect marginal effects of a one standard deviation change in the variable about its mean for continuous variables or a one unit change in the case of discrete variables.

The data show that borrowers who receive positive house price shocks are much more likely to prepay their mortgage than borrowers in locations where house prices grew less quickly than expected for all three groups of mortgages (columns 1, 3, and 5). The coefficient on annualized house price appreciation is always positive and statistically different from zero. Consistent with our model, the effect of house price appreciation on payoff are largest (both on an absolute and relative basis) for the most risky loans. For prime mortgages, a one standard deviation change in house price appreciation (1.0 percent) leads to a modest 1.8 percent increase in the payoff rate, an increase of about 5.6 percent of the mean payoff rate of 32%. For the riskier Alt-A loans, the marginal effect rises to 8.3 percent, a much larger increase of 15 percent from the mean payoff rate of 55%. Finally, for the riskiest subprime loans, the marginal effect rises to 12.6%, suggesting that a one standard deviation grown in house price appreciation can explain 18.5 percent of the variation in payoff rates. These coefficients support the conventional wisdom in the mortgage industry that house price appreciation is an economically important factor in explaining payoff rates for the riskiest mortgages.

The model above suggests that borrowers prepayment behavior should be sensitive to the unexplained portion of house price appreciation. While it is impossible to know what borrowers expect house price appreciation to be, we decompose the quarterly appreciation rate into an explained and residual component using a simple regression of current appreciation rates (2003-2007) on past appreciation rates using house price data from 1980 to 2003. The coefficient on the lagged appreciation rate is used to compute the predicted rate of house price growth. We include the predicted effect and the residual separately as dependent variables in columns 2, 4, and 6. The marginal effect of a one standard deviation in the residual is always larger than that of the explained component, with the difference between the marginal effects being statistically different from each other for the Alt-A and subprime pools. As with the model, it is shocks to house price appreciation that appears to be the dominant factor explaining the positive relationship between house price appreciation and prepayment rates. And once again we observe a monotonic relationship in which the shocks to house price appreciation have the most pronounced effect on payoffs of subprime loans and the smallest effect on payoffs of prime mortgages.

Next we show that the riskiest borrowers are the most likely to choose loans with prepayment penalties. Across the three types of pools, the evidence strongly supports our hypothesis. Only about 2.5 percent of mortgages in prime pools have prepayment penalties, while nearly 26 percent of mortgages in Alt-A pools have prepayment penalties, and about 63 percent of loans in subprime pools have prepayment penalties. Within the subprime category, Figure 4 plots the percentage of loans with prepayment penalties by three different FICO scores levels, where we have the greatest variation in credit quality. About 73 percent of borrowers with the lowest FICO scores (below 620) had prepayment penalties, while the percentage of prepayment penalties fell to 64 percent for the middle group (FICO between 620 and 679), and dropped

further to about 50 percent for subprime loans with FICO scores of 680 and above.

Of course, as noted above, many commentators believe that the riskiest borrowers are more likely to take out mortgages with prepayment penalties because they do not fully understand their contracts rather than because these borrowers receive a positive benefit from prepayment penalties. Figure 5 plots the average discount in mortgage rates enjoyed by borrowers who take out loans with prepayment penalties across three different FICO scores levels for the subprime mortgage pools. We observe that the borrowers with the lowest FICO scores receive the largest benefit from accepting prepayment penalties. Borrowers with FICO scores under 620 obtain mortgage rates of 8.0 percent, about 0.7 percent lower than the 8.7 percent mortgage rate for similar borrowers with loans that have prepayment penalties. Relative safe subprime borrowers with FICO scores above 680 have mortgage rates of 8.0 percent without a prepayment penalty and 7.5 percent with a prepayment penalty, a difference of only 0.5 percent.

We confirm these differences in means using regressions to control for other possible risk factors that might affect the mortgage rate. Table 6 presents the result of regressing the subprime coupon rates on the prepayment penalty status interacted with dummies for the three FICO score categories, while controlling for a large variety of ex-ante risk measures. We find that the borrowers with the lowest FICO scores also receive the largest benefit from accepting prepayment penalties, a mortgage rate discount of 0.68 percent. By contrast, borrowers with FICO scores over 680 obtain a mortgage rate on for purchase loans that is only 0.5 percent lower when they accept a prepayment penalty. All of these effects are individually statistically significant and the difference between the highest and lowest FICO interactions are also statistically different from each other. The reduction in mortgage rates between high and low FICO categories is even more striking for purchase mortgages (column 2). The lowest FICO borrowers pay a rate premium of about 0.8 percent, versus a premium of only 0.4 percent for the highest FICO borrowers.

Finally, we examine the most striking prediction from our model; that default rates of mortgages with prepayment penalties are lower than default rates for loans without a prepayment penalty and that the largest differences appear for the riskiest borrowers. Figure 6 reports mean default rates for mortgages with and without prepayment penalties in our three FICO categories. Our primary measure of default is whether the loan was 90 days or more past due or in some stage of the foreclosure process in the first 51 months after origination. We do not separately model foreclosures versus just severely late payments due to large differences across states in the time required to complete a foreclosure. For the riskiest mortgages, almost 18 percent of borrowers without prepayment penalties default at some point during our sample period, while only 13 percent of borrowers without prepayment rates default. Default rates appear similar for borrowers in the intermediate and high FICO categories.

Table 7 presents logit regressions for whether a subprime loan defaults on the prepayment penalty status interacted with the three FICO score categories, controlling for a variety of ex-ante risk measures. The riskiest borrowers (FICO < 620) default at a rate almost 43% higher when they do not have prepayment penalties (16.3% versus 11.4%), a difference that is economically large and statistically significant. This

difference disappears for safer borrowers with FICO scores over 620, whose default rates are lower and appear unrelated to whether or not their mortgage has a prepayment penalty. Our model suggests that the increase in default rate for the riskiest borrowers with no prepayment penalties may well be caused by the higher mortgage rates they pay. Such a result is predicted by our model, which suggests welfare losses in the form of higher defaults without prepayment penalties, with welfare losses rising with the risk of the borrower. For purchase loans, the difference in default rates of about 5 percent remains for the lowest FICO borrowers. However, the highest FICO borrowers with a prepayment penalty do appear to have a slightly higher foreclosure rate, which is counter to the predictions of our model. However, the difference in default rates between the lowest and highest credit quality borrowers is consistent with the model.

While the percentage of low FICO borrowers with fully prepayable loans is relatively low (less than 30 percent), the question naturally arises as to why low credit quality borrowers would not have prepayment penalties if the higher mortgage rates often lead to greater defaults. Our earlier discussion suggests that some borrowers with a high likelihood of moving might accept the higher risk of default to compensate for avoiding being locked-into their house. For all borrowers, the impact of being locked-in may impose some additional costs that present trade-offs versus the rate benefit of a prepayment penalty. The data do not allow us to understand the specific terms of the prepayment penalty. Some mortgages have provisions for prepayment penalties to be waived if the owner sells the house, while other prepayment penalties are enforced in all circumstance. Although outside the scope of our model, if a prepayment penalty limits the possibility of a borrower selling and moving in response to a negative economic shock, prepayment penalties can present an offsetting cost. While the net effect of prepayment penalties is to reduce defaults for the riskiest borrowers, they may have some smaller offsetting costs for relatively safer borrowers. Future work will need to explore the small increase in foreclosure rates for the best borrowers with prepayment penalties.

## 8 Concluding Remarks

Critics of subprime mortgage lending argue that prepayment penalties unfairly lock-in risky borrowers to mortgages with high interest payments, accusing mortgage originators of predatory lending to naive borrowers who do not fully understand mortgage terms. This paper shows that in a competitive lending model the refinancing penalties are welfare improving, and that they are particularly beneficial to riskier borrowers in terms of lower mortgage rates and reduced defaults. The empirical work supports key predictions of our model.

These results suggest that the legislation banning refinancing penalties might have unintended consequences. Instead of protecting would-be homeowners from predatory lending, the new law might end up protecting them from credit.

Our analysis is based on the assumption of that borrowers can fully understand the mortgage lending terms. Evidence from other developed countries shows that mortgages with prepayment penalties are the

dominant type of contract around the world, with the primary exceptions being the US and the Netherlands. Thus, we do not believe that such contracts are inherently predatory. Of course, for borrowers lacking self control, any contract that involved future commitments may be prone to abuse. However, policy makers considering banning refinancing penalties should recognize the potential economic costs that such legislation could imply in terms of market exclusion and higher interest rates for the most risky borrowers.

# Appendix

## Proof of Proposition 1

First we show that the solution to (3)-(6) is concave in  $S$ . Consider a function

$$F(S, \bar{\phi}) = \bar{U}(S, \bar{\phi}) - S,$$

where  $\bar{U}(S, \bar{\phi})$  is a solution to (3)-(6). From (3)-(6) we have that

$$\begin{aligned} \gamma F(S, \bar{\phi}) &= \mu + \theta - \bar{\phi} + (rS + \mu - \bar{\phi}) F'(S, \bar{\phi}) - (\gamma - r)W + \frac{1}{2}\sigma^2 F''(S, \bar{\phi}) \text{ for } S \in [0, S^1(\bar{\phi})], \\ F(0, \bar{\phi}) &= A, \\ F'(S, \bar{\phi}) &= 0 \text{ for } S \geq S^1(\bar{\phi}), \\ F''(S, \bar{\phi}) &= 0 \text{ for } S \geq S^1(\bar{\phi}). \end{aligned}$$

As the solution  $F$  is smooth we can differentiate it with respect to  $S$  to obtain:

$$(\gamma - r)F'(S, \bar{\phi}) = (rS + \mu - \bar{\phi}) F''(S, \bar{\phi}) - (\gamma - r) + \frac{1}{2}\sigma^2 F'''(S, \bar{\phi}) \text{ for } S \in [0, S^1(\bar{\phi})].$$

Evaluating the above equation at  $S = S^1(\bar{\phi})$  yields

$$F'''_-(S^1(\bar{\phi}), \bar{\phi}) = \frac{2(\gamma - r)}{\sigma^2} > 0.$$

Note that as  $F''(S^1(\bar{\phi}), \bar{\phi}) = 0$  and we have that  $F'''_-(S^1(\bar{\phi}), \bar{\phi}) > 0$  it implies that there exists  $\varepsilon > 0$  such that  $F'' < 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ . Also as  $F'(S^1(\bar{\phi}), \bar{\phi}) = 0$  and  $F''(S^1(\bar{\phi}), \bar{\phi}) < 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$  it implies that  $F' > 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ .

From (3) we have that

$$F''(S, \bar{\phi}) = \frac{2[\gamma F(S, \bar{\phi}) - [\mu + \theta - \bar{\phi} - (\gamma - r)S] - (rS + \mu - \bar{\phi}) F'(S, \bar{\phi})]}{\sigma^2}. \quad (7)$$

As  $\bar{\phi} \leq \theta$  and as by assumption  $\theta \leq \mu$  we have that  $\mu - \bar{\phi} \geq 0$ . Hence from (7) whenever: (i)  $\gamma F(S, \bar{\phi}) < [\mu + \theta - \bar{\phi} - (\gamma - r)S]$  and (ii)  $F'(S, \bar{\phi}) > 0$  holds it follows that  $F'' < 0$ . Now since  $\gamma F(S^1(\bar{\phi}), \bar{\phi}) = \mu + \theta - \bar{\phi} - (\gamma - r)S^1(\bar{\phi})$  and  $F' > 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$  it follows that these conditions are satisfied over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ . Moreover, (i) will hold for any  $S \in [0, S^1(\bar{\phi})]$  as long as  $F$  remains strictly increasing, i.e. as long as  $F' > 0$ .

Now suppose by contradiction that  $F' \leq 0$  for some  $S \leq S^1(\bar{\phi}) - \varepsilon$ , and let  $\tilde{S} = \sup \{S \leq S^1(\bar{\phi}) - \varepsilon \mid F' \leq 0\}$ . Then it follows that  $F'(\tilde{S}, \bar{\phi}) = 0$ , and that for all  $S \in (\tilde{S}, S^1(\bar{\phi}))$  we have that  $F' > 0$  and so (i) and (ii) holds. But this implies that  $F'' < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$ . From the Fundamental Theorem of Calculus it

follows that:

$$S'(S^1(\bar{\phi}), \bar{\phi}) = F'(\tilde{S}, S^1(\bar{\phi})) + \int_{\tilde{S}}^{S^1(\bar{\phi})} F''(S, \bar{\phi}) dS,$$

which given that  $S'(S^1(\bar{\phi}), \bar{\phi}) = 0$  implies that

$$F'(\tilde{S}, S^1(\bar{\phi})) = - \int_{\tilde{S}}^{S^1(\bar{\phi})} F''(S, \bar{\phi}) dS.$$

As  $F'' < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$  the above implies that  $F'(\tilde{S}, S^1(\bar{\phi})) > 0$ , which is a contradiction. Hence we have that  $F' > 0$  for  $S \in (0, S^1(\bar{\phi}))$  and hence (i) and (ii) holds and so  $F'' < 0$  for  $S \in (0, S^1(\bar{\phi}))$ . But this implies that  $\bar{U}'(S, \bar{\phi}) > 1$  and  $\bar{U}''(S, \bar{\phi}) < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$ .

We note that Conditions (5)-(6) imply that

$$\gamma \bar{U}(S^1(\bar{\phi}), \bar{\phi}) = \theta + \mu - \bar{\phi} + rS^1(\bar{\phi}) \quad (8)$$

The borrower's wealth evolves according to

$$dS_t = (rS_t + \mu - \bar{\phi}) dt + \sigma dZ_t - dC_t$$

For an arbitrary feasible strategy  $(C, S)$ , consider

$$G_t \equiv \int_0^t e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma t} \bar{U}(S_t, \bar{\phi}), \quad (9)$$

where function  $\bar{U}$  satisfies all the conditions outlined in Proposition 1. We will show that  $G_t$  is a supermartingale. Differentiating (9) with respect to  $t$  and using Ito's lemma gives

$$\begin{aligned} e^{\gamma t} dG_t &= [-\gamma \bar{U}(S_t, \bar{\phi}) + \theta + (rS_t + \mu - \bar{\phi}) \bar{U}'(S_t, \bar{\phi}) + 0.5\sigma^2 \bar{U}''(S_t, \bar{\phi})] dt \\ &\quad + (1 - \bar{U}'(S_t, \bar{\phi})) dC_t + \bar{U}'(S_t, \bar{\phi}) \sigma dZ_t. \end{aligned} \quad (10)$$

When  $S_t \leq S^1(\bar{\phi})$ , the first term in the right-hand side of (10) is zero, because of (3). When  $S_t > S^1(\bar{\phi})$ , the first term is negative, because of (5), (6), (8) and the fact  $\gamma > \rho$ . Since  $\bar{U}'(S_t, \bar{\phi}) \geq 1$  and  $dC_t \geq 0$ ,  $G_t$  is a supermartingale.

$$\begin{aligned} E \left[ \int_0^\tau e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma \tau} A \right] &= E \left[ \int_0^\tau e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma \tau} S_\tau \right] \\ &= E[G_\tau] \leq G_0 = \bar{U}(S_0, \bar{\phi}) \end{aligned} \quad (11)$$

Thus, the agent's payoff associated with strategy  $(C, W)$  is less than or equal to  $\bar{U}(S_0, \bar{\phi})$ .

If the agent has zero consumption whenever  $S_t \in [0, S^1(\bar{\phi})]$  and consumes  $S_t - S^1(\bar{\phi})$  immediately whenever  $S_t > S^1(\bar{\phi})$ , then  $G_t$  is a martingale and (11) holds with equality. Hence, this is the optimal strategy, which results in  $\bar{U}(S_0, \bar{\phi})$  payoff to the agent.  $\square$

## Proof of Proposition 2

First we prove the following Lemma.

**Lemma 3** *The maximum savings level of the borrower,  $S^1(\bar{\phi}^*)$ , is strictly increasing in the competitive mortgage repayment rate on the FRM with no refinancing  $\bar{\phi}^*$ .*

**Proof** Differentiating equation (8) with respect to  $\bar{\phi}$  and taking into account (5) gives

$$\frac{\partial S^1(\bar{\phi}^*)}{\partial \bar{\phi}^*} = -\frac{1 + \frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*}}{\gamma - r}.$$

According to Feynman-Kac formula,

$$\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} = -E \left[ \int_0^{\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} e^{-\gamma t} \frac{\partial \bar{U}(S_t, \bar{\phi}^*)}{\partial S_t} dt \mid S_0 = S^1(\bar{\phi}^*) \right].$$

Since  $\frac{\partial \bar{U}(S_t, \bar{\phi}^*)}{\partial S_t} \geq 1$ , we can write that

$$\begin{aligned} -\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} &\geq E \left[ \int_0^{\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} e^{-\gamma t} dt \right] \\ &= \frac{1}{\gamma} \left( 1 - E \left[ e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} \right] \right). \end{aligned}$$

On the other hand, since  $S_0 \leq S^1(\bar{\phi}^*)$  and  $r < \gamma$  we can write that

$$\begin{aligned} P &= V(S_0, \bar{\phi}^*) \leq V(S^1(\bar{\phi}^*), \bar{\phi}^*) = \frac{\bar{\phi}^*}{r} - \left( \frac{\bar{\phi}^*}{r} - L \right) E \left[ e^{-r\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} \right] \\ &< \frac{\bar{\phi}^*}{r} - \left( \frac{\bar{\phi}^*}{r} - L \right) E \left[ e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} \right], \end{aligned}$$

which gives

$$E \left[ e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} \right] \leq \frac{\frac{\bar{\phi}^*}{r} - P}{\frac{\bar{\phi}^*}{r} - L}.$$

Hence,

$$\begin{aligned} -\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} &\geq \frac{1}{\gamma} \left( 1 - \frac{\frac{\bar{\phi}^*}{r} - P}{\frac{\bar{\phi}^*}{r} - L} \right) \\ &= 1 + \frac{P - \left( \gamma \frac{\bar{\phi}^*}{r} + (1 - \gamma)L \right)}{\gamma \left( \frac{\bar{\phi}^*}{r} - L \right)}. \end{aligned}$$

Noting that  $\bar{\phi}^* \leq \theta$  (Lemma 1), and as  $\theta \leq \frac{r[P - (1 - \gamma)L]}{\gamma}$  (Assumption 1) we have that  $P \geq \gamma \frac{\bar{\phi}^*}{r} + (1 - \gamma)L$ . But this implies that  $\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} < -1$  and so  $\frac{\partial S^1(\bar{\phi}^*)}{\partial \bar{\phi}^*} > 0$ . ■

Now let's take any  $S_0 \leq S^1(\bar{\phi}^*(S_0))$ . Consider  $S'_0 < S_0$ . Let's suppose that by contradiction  $\bar{\phi}^*(S'_0) < \bar{\phi}^*(S_0)$ . But this and Lemma 3 imply that  $S^1(\bar{\phi}^*(S'_0)) < S^1(\bar{\phi}^*(S_0))$ , which implies that  $\tau(S_0, \bar{\phi}^*(S_0)) > \tau(S'_0, \bar{\phi}^*(S'_0))$  (as both  $S'_0 < S_0$  and  $S^1(\bar{\phi}^*(S'_0)) < S^1(\bar{\phi}^*(S_0))$ ). But this implies that

$$P = \underbrace{\frac{\bar{\phi}^*(S_0)}{r} - E \left[ e^{-r\tau(S_0, \bar{\phi}^*(S_0))} \left( \frac{\bar{\phi}}{r} - L \right) | \mathcal{F}_0 \right]}_{V_0(S_0, \bar{\phi}^*(S_0))} > \underbrace{\frac{\bar{\phi}^*(S'_0)}{r} - E \left[ e^{-r\tau(S'_0, \bar{\phi}^*(S'_0))} \left( \frac{\bar{\phi}}{r} - L \right) | \mathcal{F}_0 \right]}_{V_0(S'_0, \bar{\phi}^*(S'_0))}, \quad (12)$$

which contradicts  $V_0(S'_0, \bar{\phi}^*(S'_0)) = P$ . Therefore we have that the competitive mortgage rate on FRM with no refinancing,  $\phi^*$ , is strictly decreasing on  $[\underline{S}, \bar{S}]$ , where  $\bar{S}$  is the asset level of the most creditworthy borrower. Moreover

$$\bar{S} = \inf_{S \geq \underline{S}} \{S^1(\bar{\phi}^*(S))\},$$

as we have that  $\frac{d\bar{\phi}^*(S)}{dS} < 0$  for  $S \in [\underline{S}, \bar{S}]$  and  $\frac{\partial S^1(\bar{\phi}^*)}{\partial \bar{\phi}^*} > 0$  (Lemma 3). □

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**TABLE 2: Variable Description**

<b>Pay Off in 17 Quarters</b>	A variable that is 1 if the loan pays off in full (loan balance goes to 0) in the 17Q following June 2003 and 0 otherwise.
<b>90 Days Past Due in 17 Quarters</b>	A variable that is 1 if the loan becomes at least 90 days delinquent on a payment in the 17 quarters following June 2003 and 0 otherwise.
<b>17 Quarter HPI Growth</b>	This variable is the quarterly, real HPI growth from Q3 2003 to Q2 2007 in the MSA of the loan.
<b>Predicted, Residual 17 Quarter HPI Growth</b>	The results of an MSA level regression of the quarterly, real HPI Growth from Q3 2003 to Q2 2007 on the quarterly, real HPI growth from Q1 1980 to Q2 2003.
<b>Prepayment Penalty</b>	A variable that is 1 if the loan contains a prepayment penalty of any sort and 0 otherwise.
<b>Interest Rate Buydown</b>	A variable that is 1 if the borrower bought down the coupon rate using an upfront payment and 0 otherwise.
<b>Coupon Rate</b>	The monthly interest rate charged to the borrower.
<b>FICO Score</b>	The credit score of the borrower using the FICO credit profile.
<b>Origination Amount</b>	The origination amount of the loan in \$100,000's.
<b>Combined LTV</b>	A variable that is the Loan-to-Value ratio calculated as the sum of the value of all liens against the home over the value of the home.
<b>Loan Purpose</b>	A variable that is either <i>For Purchase</i> , <i>Cash Out Refinance</i> or <i>No Cash Out Refinance</i> . <i>For Purchase</i> is the reference variable in the following regressions
<b>Loan Documentation Level</b>	A variable that is either <i>Full Documentation</i> , <i>Low Documentation</i> or <i>No Documentation</i> . <i>Full Documentation</i> is the reference variable in the following regressions.

**TABLE 3: Fixed Rate Mortgages without Prepayment Penalties  
Summary Statistics**

	Prime Pools			Alt A Pools			Subprime Pools		
	Mean	Med	Std Dev	Mean	Med	Std Dev	Mean	Med	Std Dev
Pay Off in 17 Quarters (d)	0.32	0.0	0.47	0.55	1.0	0.50	0.68	1.0	0.47
90 Days Past Due in 17 Quarters (d)	0.003	0.0	0.052	0.033	0.0	0.18	0.097	0.0	0.30
17 Quarter HPI Growth	0.017	0.016	0.009	0.015	0.017	0.010	0.013	0.013	0.010
Predicted 17 Quarter HPI Growth	0.013	0.013	0.002	0.011	0.011	0.001	0.010	0.010	0.001
Residual 17 Quarter HPI Growth	0.004	0.002	0.009	0.004	0.005	0.010	0.002	0.002	0.010
Coupon Rate	5.48	5.50	0.35	6.13	6.00	0.76	8.34	7.85	2.07
Interest Rate Buydown (d)	0.066	0.0	0.248	0.038	0.0	0.191			
FICO Score	740	749	42	710	710	48	658	661	49
Origination Amount (in \$100K)	4.80	4.42	1.63	2.54	2.00	1.78	1.27	0.87	1.13
Origination Amount Squared	25.69	19.54	20.84	9.61	4.00	18.69	2.89	0.76	6.34
Combined Loan-to-Value	64.7	67.6	15.7	77.9	80.0	17.9	89.0	90.0	17.2
Cash Out Refinance (d)	0.19	0.0	0.39	0.37	0.0	0.48	0.52	1.0	0.50
No Cash Out Refinance (d)	0.63	1.0	0.48	0.26	0.0	0.44	0.14	0.0	0.34
Low Documentation (d)	0.28	0.0	0.45	0.55	1.0	0.50	0.37	0.0	0.48
No Documentation (d)	0.07	0.0	0.25	0.09	0.0	0.28	0.02	0.0	0.14
Observations	10,630			9,601			8,715		

Loan refers to a fixed rate mortgage without prepayment penalties on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data and an origination amount greater than \$25,000. In addition, the subprime pools are restricted to those mortgages with a FICO between 550 and 750. HPI Growth is an MSA level statistic originating from the Office of Federal Housing Enterprise Oversight's house price index. Predicted and residual 17 quarter HPI Growth represent the outcome of an MSA-level regression of HPI Growth Q2 2003 - Q2 2007 on HPI Growth Q1 1980 - Q1 2003. Residual 17 Quarter HPI Growth has a non-zero mean because the number of loans within each MSA varies. Pay Off is defined as completely paying off the principle balance of a loan. Discrete variables are denoted by (d).

**TABLE 4: Fixed Rate Mortgages without Prepayment Penalties**  
**Logit Regression of Pay Off on House Price Growth**

	Prime		Alt A		Subprime	
<b>Dependent variable: Pays Off</b>	<b>0.32</b>		<b>0.55</b>		<b>0.68</b>	
17 Quarter HPI Growth	0.018**		0.083**		0.126**	
	(2.54)		(7.35)		(10.41)	
Predicted 17Q HPI Growth		0.016*		0.039**		0.071**
		(1.95)		(3.17)		(7.30)
Residual 17Q HPI Growth		0.018**		0.080**		0.107**
		(2.40)		(7.09)		(10.15)
Interest Rate Buydown (d)	-0.017	-0.018	0.028	0.026		
	(-0.70)	(-0.73)	(1.09)	(1.01)		
Coupon Rate	0.058**	0.057**	0.105**	0.103**	0.116**	0.107**
	(10.46)	(10.31)	(9.26)	(9.13)	(10.91)	(9.58)
FICO Score < 620 (d)	-0.049	-0.042	0.081**	0.083**	-0.038**	-0.028
	(-0.69)	(-0.60)	(2.39)	(2.52)	(-2.07)	(-1.48)
620 < FICO Score < 680 (d)	0.099**	0.060**	0.078**	0.066**	0.007	0.017
	(6.55)	(4.13)	(6.55)	(4.81)	(0.54)	(1.39)
Origination Amount (in \$100K)	-0.059**	-0.059**	0.041	0.020	0.106**	0.070**
	(-2.73)	(-2.62)	(1.47)	(0.82)	(3.70)	(2.18)
Origination Amount Squared	0.042**	0.042*	-0.037*	-0.037	-0.089**	-0.070**
	(1.96)	(1.86)	(-1.65)	(-1.33)	(-4.50)	(-3.33)
CLTV > 97 (d)	0.112	0.122	0.072**	0.084**	0.004	0.009
	(0.79)	(0.87)	(2.71)	(3.12)	(0.20)	(0.41)
90 < CLTV < 97 (d)	0.121**	0.119**	0.105**	0.113**	0.013	0.018
	(5.48)	(5.34)	(5.47)	(5.98)	(0.65)	(0.72)
80 < CLTV < 90 (d)	0.069**	0.070**	0.074**	0.079**	-0.016	0.002
	(3.80)	(3.80)	(4.59)	(4.92)	(-0.88)	(0.10)
Cash Out Refi (d)	0.063**	0.058**	-0.016	-0.020	-0.091**	-0.081**
	(3.27)	(3.07)	(-0.97)	(-1.15)	(-5.58)	(-5.57)
No Cash Out Refi (d)	0.055**	0.052**	-0.066**	-0.065**	-0.111**	-0.103**
	(4.32)	(4.19)	(-3.17)	(-3.06)	(-5.56)	(-5.29)
Low Doc (d)	0.016	0.016	0.076**	0.072**	0.009	0.010
	(1.36)	(1.31)	(5.24)	(5.01)	(0.61)	(0.81)
No Doc (d)	-0.009	-0.007	0.071**	0.062**	-0.007	-0.001
	(-0.45)	(-0.32)	(4.04)	(3.57)	(-0.15)	(-0.02)
Observations	10,630	10,630	9,601	9,601	8,715	8,715

This table reports the marginal effects of a Logit regression. For discrete variables we report the effect of changing from 0 to 1. For continuous variables, we report the effect of increasing 1 std. dev. from the mean. Loan refers to a fixed rate mortgage without prepayment penalties on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data and an origination amount greater than \$25,000. In addition, the subprime pools are restricted to those mortgages with a FICO between 550 and 750. Pay Off is defined as completely paying off the principle balance of a loan. Discrete variables are denoted by (d). T-statistics reported in parentheses.

**TABLE 5: Fixed Rate Subprime Mortgages  
Summary Statistics**

	Full Sample			For Purchase Loans		
	Mean	Med	Std Dev	Mean	Med	Std Dev
Pay Off in 17 Quarters (d)	0.67	1.0	0.47	0.74	1.0	0.44
90 Days Past Due in 17 Quarters (d)	0.10	0.0	0.30	0.13	0.0	0.33
17 Quarter HPI Growth	0.015	0.017	0.011	0.016	0.017	0.011
Coupon Rate	8.00	7.50	1.83	8.72	8.25	2.02
Prepayment Penalty (d)	0.63	1.0	0.48	0.55	1.0	0.50
Fico Score	646	645	48	657	656	46
Origination Amount (in \$100K)	1.41	1.12	1.07	1.22	0.86	1.05
Origination Amount Squared	3.13	1.25	5.65	2.59	0.75	5.32
Combined Loan-to-Value	85.5	88.6	17.2	93.9	100.0	9.7
For Purchase Loan (d)	0.28	0.0	0.45	1.00	1.0	0.00
Refinance, Cash Out (d)	0.58	1.0	0.49	0.00	0.0	0.00
Refinance, No Cash Out (d)	0.14	0.0	0.35	0.00	0.0	0.00
Full Documentation (d)	0.66	1.0	0.47	0.51	1.0	0.50
Low Documentation (d)	0.33	0.0	0.47	0.46	0.0	0.50
No Documentation (d)	0.01	0.0	0.12	0.03	0.0	0.16
<b>Observations</b>	<b>23,633</b>			<b>6,703</b>		

Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. HPI Growth is an MSA level statistic originating from the Office of Federal Housing Enterprise Oversight's house price index. Discrete variables are denoted by (d).

**TABLE 6: Fixed Rate Subprime Mortgages**  
**OLS Regression of Coupon Rate on Prepayment Penalty Status**

	<b>Full Sample</b>	<b>For Purchase Loans</b>
<b>Dependent Variable: Coupon Rate</b>	<b>8.00 (1.83)</b>	<b>8.72 (2.02)</b>
Prepayment Penalty [PP PEN] at FICO < 620 (d)	-0.68** (-18.79)[8.691]	-0.80** (-7.76)[9.977]
PP PEN at 620 <= FICO < 680 (d)	-0.68** (-16.31)[8.467]	-0.59** (-8.47)[9.318]
PP PEN at 680 <= FICO (d)	-0.51** (-9.06)[7.977]	-0.44** (-4.39)[8.07]
17 Quarter HPI Growth	0.002 (0.08)	0.12** (2.15)
CLTV > 97 (d)	2.88** (57.18)	2.11** (22.52)
90 < CLTV < 97 (d)	0.79** (36.58)	1.03** (16.23)
80 < CLTV < 90 (d)	0.44** (29.77)	0.50** (9.75)
Origination Amount (in \$100K)	-1.52** (-15.25)	-1.93** (-10.00)
Origination Amount Squared	1.01** (11.12)	1.25** (6.02)
Low Documentation (d)	0.45** (15.76)	0.78** (11.19)
No Documentation (d)	1.02** (9.54)	1.93** (15.93)
For Purchase Loan [Purchase] (d)	0.03 (0.51)	
Purchase * CLTV > 97 (d)	-0.71** (-6.42)	
Purchase * 90 < CLTV < 97 (d)	0.18** (3.04)	
Purchase * 80 < CLTV < 90 (d)	-0.05 (-0.94)	
<b>Observations</b>	<b>23,633</b>	<b>6,703</b>

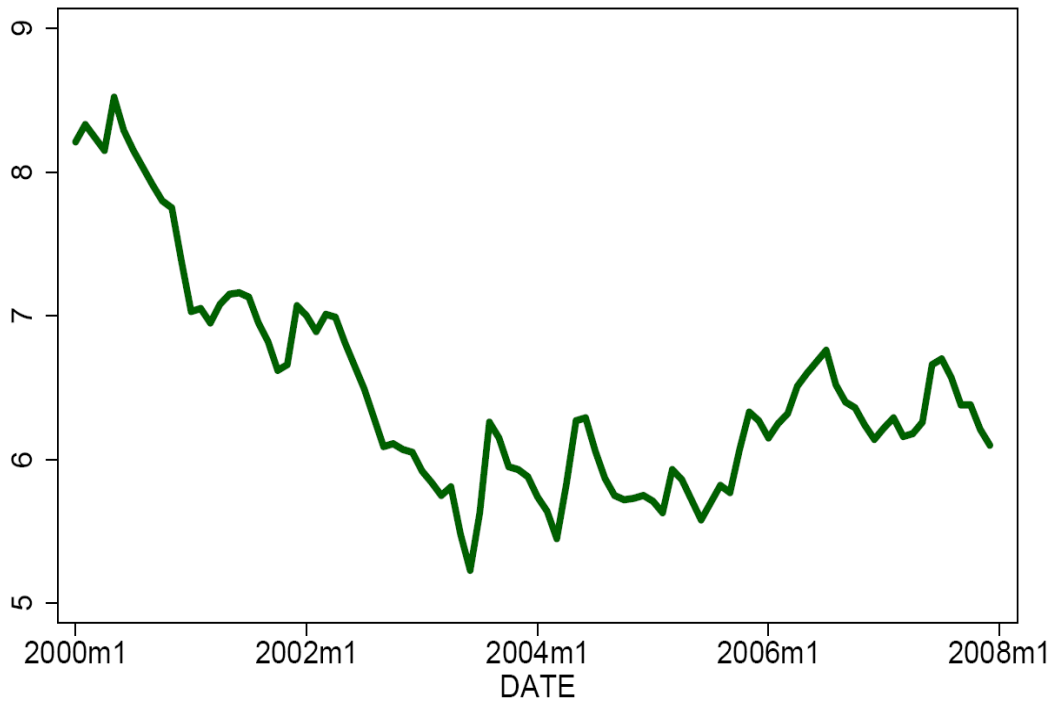
This table reports the results of an OLS regression. For discrete variables we report the effect of changing from 0 to 1 (i.e. coefficient). For continuous variables, we report the effect of increasing 1 std. dev. Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. Discrete variables are denoted by (d). T-statistics reported in parentheses. Baseline predicted probability for group reported in square brackets.

**TABLE 7: Fixed Rate Subprime Mortgages**  
**Logit Regression of 90 Days Past Due on Prepayment Penalty Status**

	Full Sample	For Purchase Loans
<b>Dependent Variable: 90 Days Past Due</b>	<b>0.097</b>	<b>0.126</b>
Prepayment Penalty [PP PEN] at FICO < 620 (d)	-0.049** (-5.04)[0.163]	-0.054** (-2.38)[0.250]
PP PEN at 620 <= FICO < 680 (d)	-0.004 (-0.74)[0.080]	-0.012 (-1.06)[0.116]
PP PEN at 680 <= FICO (d)	0.004 (0.9.1)[0.041]	0.023** (2.63)[0.042]
17Q Cumulative HPI Growth	-0.039** (-13.92)	-0.056** (-13.49)
CLTV > 97 (d)	0.074** (6.17)	0.107** (4.56)
90 < CLTV < 97 (d)	0.055** (6.13)	0.124** (3.78)
80 < CLTV < 90 (d)	0.033** (3.70)	0.119** (3.13)
Origination Amount (in \$100K)	-0.017** (-2.93)	-0.014 (-1.53)
Origination Amount Squared	0.011** (3.30)	0.011* (1.79)
Low Documentation (d)	0.018** (3.90)	0.011 (1.27)
No Documentation (d)	0.029** (2.06)	0.041* (1.70)
For Purchase Loan [Purchase] (d)	-0.005 (-0.27)	
Purchase * CLTV > 97 (d)	0.024 (1.03)	
Purchase * 90 < CLTV < 97 (d)	0.038 (1.44)	
Purchase * 80 < CLTV < 90 (d)	0.052 (1.64)	
Observations	23,598	6,684

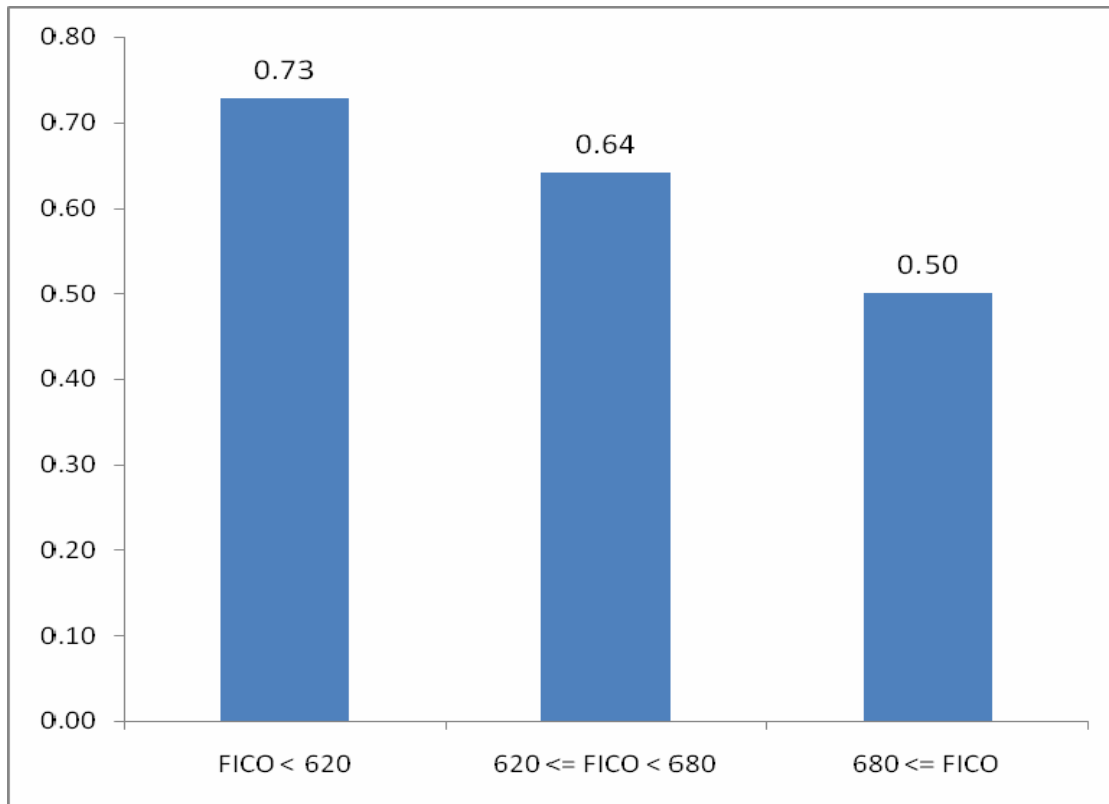
This table reports the marginal effects of a Logit regression. Coefficients on discrete variables represent the effect of moving from 0 to 1. Coefficients on continuous variable represent the effect of moving 1 standard deviation around the mean. Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. Discrete variables are denoted by (d). T-statistics reported in parentheses. Without prepayment penalty predicted probability for group reported in square brackets.

**Figure 3: 30 Year Fixed Rate Conforming Mortgages  
Average Mortgage Rate: Jan. 2000 to Dec. 2007**



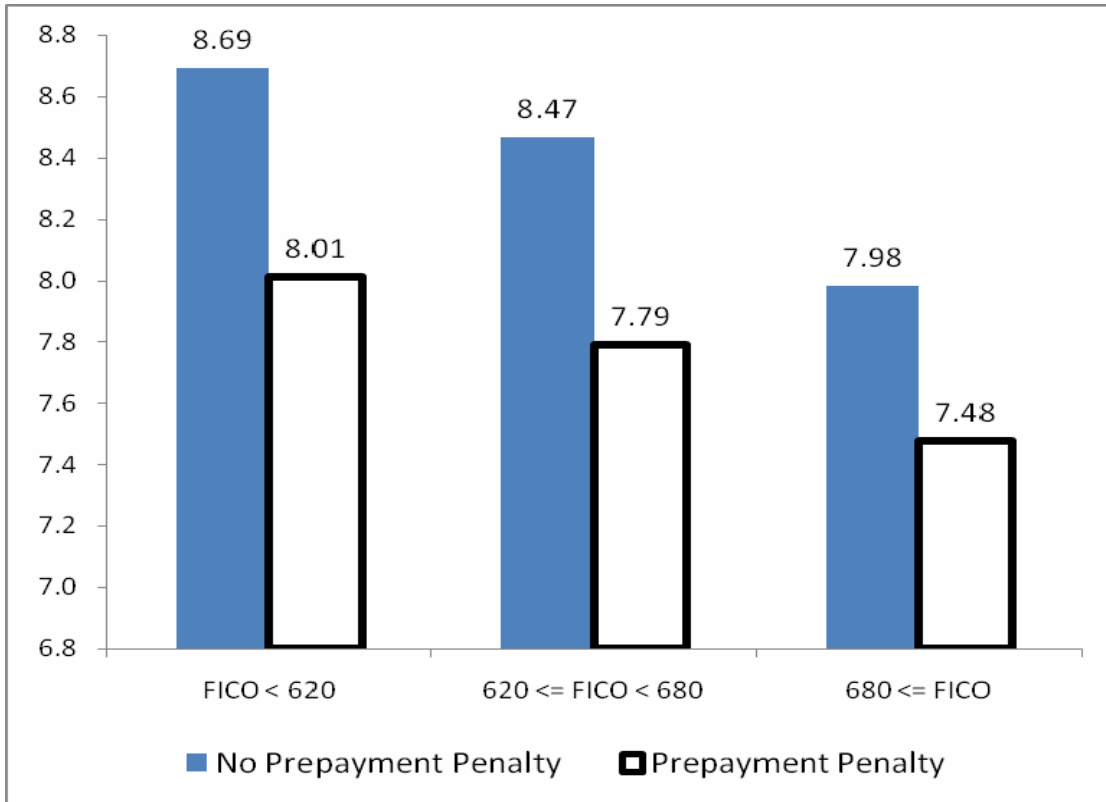
Mean 30 Yr. Mortgage Rate  
Source: Federal Reserve Economic Data  
Sample: Fixed rate mortgages originated between 2000 and 2007

**Figure 4: Fixed Rate Subprime Mortgages**  
**Fraction of Loans with Prepayment Penalties by FICO score**



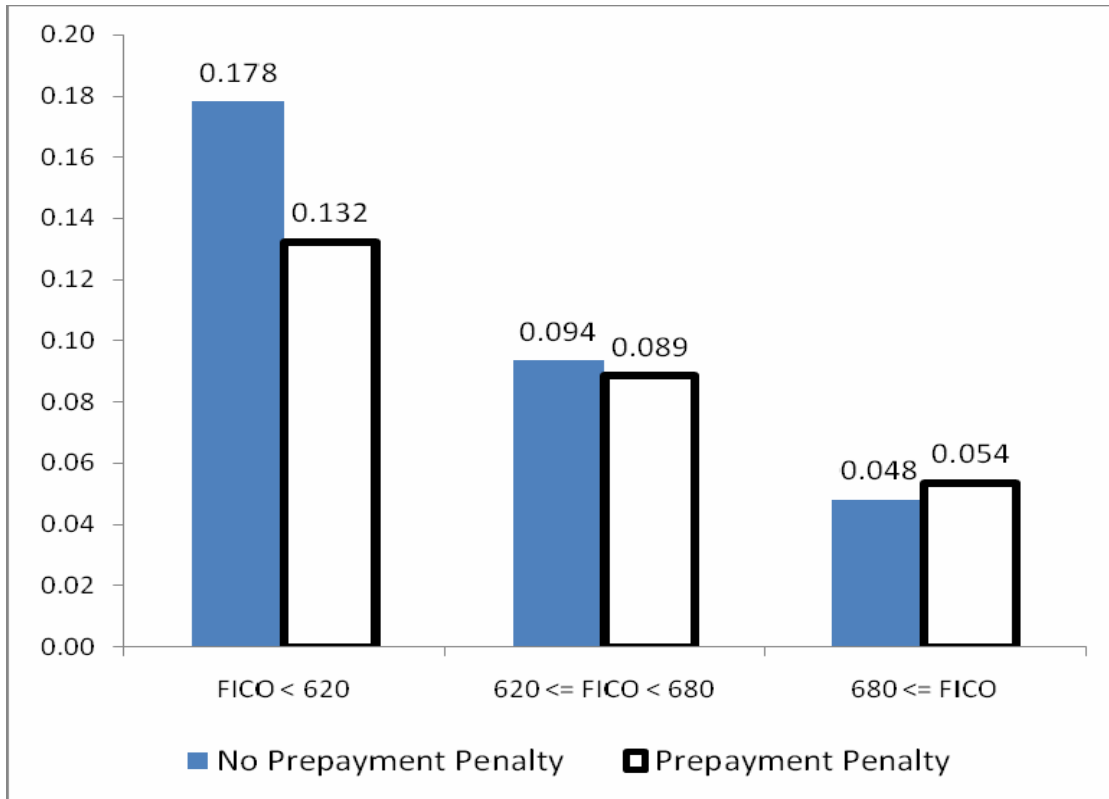
Bars represent the fraction of loans that have prepayment penalties. Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. Percentage point discount is reported above each column.

**Figure 5: Fixed Rate Subprime Mortgages**  
**Coupon rate discount for incurring a prepayment penalty**



Bars represent the average mortgage rate charged to borrowers with and without prepayment penalties. Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. Percentage point discount is reported above each column.

**Figure 6: Fixed Rate Subprime Mortgages**  
**Reduction in delinquency rate associated with incurring a prepayment penalty**



Bars represent the 90 day delinquency rate on a mortgage by prepayment penalty status. Loan refers to a fixed rate mortgage on an owner occupied home in the LoanPerformance database originated in June 2003 with HPI Growth data, a FICO between 550 and 700, and an origination amount greater than \$25,000. Percentage point discount is reported above each column.